1. Compute the following limits.

\[
\begin{align*}
\text{(a) } & \lim_{x \to \infty} \frac{\ln x}{e^{2x}} & \quad \text{[3]} \quad \text{(b) } & \lim_{x \to 2} (x - 1)^{\frac{1}{x-2}} & \quad \text{[3]} \quad \text{(c) } & \lim_{x \to 6} \frac{\sqrt{x^2 - 2} - 2}{\sqrt{x - 6}} & \quad \text{[3]}
\end{align*}
\]

2. Compute the indicated derivatives. You do not need to simplify your answer.

\[
\begin{align*}
\text{(a) } & \text{ If } y = \frac{\cos x^2}{\sin x}, \text{ find } y'. & \quad \text{[4]} \quad \text{(b) } & \text{ If } y = (1 - 3x)e^x, \text{ find } \frac{dy}{dx}. & \quad \text{[4]}
\end{align*}
\]

3. (b) Let \( f(x) = \frac{3x + 1}{x - 2} \). Use the DEFINITION of the derivative to find \( f'(1) \).

4. (c) Find the equation of the tangent line to \( y = f(x) \) at \( x = 1 \).

6. (c) Find the value of \( b \) so that the function

\[
f(x) = \begin{cases} 
x^3 + bx + 3 & \text{if } x \leq 2 \\
be^{x-2} & \text{if } x > 2
\end{cases}
\]

is continuous everywhere. Justify your answer.

10. A rock thrown vertically upward from the surface of the moon at a velocity of 24 m/sec reaches a height of \( s = 24t - 0.8t^2 \) metres in \( t \) sec.

2. (a) Find the rock's velocity and acceleration at time \( t \).

2. (b) How long does it take the rock to reach its highest point?

2. (c) How high does the rock go?

2. (d) How long is the rock aloft?

6. 11. Find the equation of the tangent line to the curve \( x^3 + y^3 - 9xy = 0 \) at the point \( (2, 4) \).

8. 13. Consider a cube of variable size (the edge length is increasing). Assume that the volume of the cube in increasing at the rate of 10 cm\(^3\)/minute. How fast is the surface area increasing when the edge length is 8 cm?
1. (a) Find \( f'(x) \), but do not simplify, given \( f(x) = \frac{\sin(x) \cos(x)}{1 + e^{3x}} \).

(b) Find \( \frac{dy}{dx} \), but do not simplify, given \( y = \sin \left( \sqrt{1 + x^4 - x^2} \right) \).

2. Find the exact values of these limits. Show your work.

(a) \( \lim_{x \to 2} \frac{x^3 - 8}{x^2 - 4} \).

(b) \( \lim_{x \to 0} \frac{x}{\sqrt{1 + 7x} - 1} \).

5. Ocean water absorbs sunlight, so that the light intensity \( L(x) \) at depth \( x \) below the surface of the ocean satisfies the differential equation

\[
\frac{dL}{dx} = -kL
\]

thus

\[
A = A_0 e^{-kL}
\]

for some constant \( k \). Experienced divers in the waters off Haida Gwaii know that at a depth of 6 m, the light intensity is half its value at the surface. They can work without artificial light down to a depth where the light intensity is one-tenth of its value at the surface. How deep is this?

6. Consider the curve \( y = \sqrt{c^2 + x^2} \), where \( c \) is a constant obeying \( c > 1 \).

(a) Show that the curve is concave up and make a rough sketch, clearly labelling all intercepts.

(b) Find the \((x, y)\)-coordinates of each point on the curve from which the tangent line passes through the point \((0, 1)\).

(b) Use limit-evaluation methods and the definition in part (a) to calculate \( f'(1) \) for the function

\[
f(x) = \frac{1}{x + 3}.
\]

[Do not use differentiation rules in this part.]

8. A fugitive whose height is 2 meters runs straight away from a searchlight mounted 10 meters above a point \( O \) on the ground. The ground is horizontal; the runner's speed is 8 meters per second. How fast is the shadow of the runner's head moving along the ground . . .

(a) when the runner is 15 meters from \( O \)?

(b) when the runner is 25 meters from \( O \)?

(c) when the runner is \( x \) meters from \( O \) (as a function of \( x \))?
1. For each of the following evaluate the limit if it exists or otherwise explain why it does not exist.

(a) \( \lim_{x \to -2} \frac{\frac{1}{x} - \frac{1}{2}}{x - 2} \)  
(b) \( \lim_{x \to -4} \frac{|x + 4|}{x + 4} \)  
(c) \( \lim_{x \to \infty} \frac{x}{\sqrt{1 + 2x^2}} \)

2. Differentiate each of the following with respect to \( x \).

(a) \( y = e^{4x} \)  
(b) \( y = \frac{3x - 5}{x^2 + 1} \)  
(c) \( y = x \ln(x^2 + 4) \)  
(d) \( y = \sin(x^2) - \sin^2(x) \)

3. Use the definition of derivative to find \( f'(x) \) where \( f(x) = \sqrt{2x + 1} \).

7. Let

\[ f(x) = e^{1/x} \quad f'(x) = -\frac{e^{1/x}}{x^2} \quad f''(x) = \frac{e^{1/x}(2x + 1)}{x^4} \]

(a) What is the domain of \( f \)?
(b) Determine any points of intersection of the graph of \( f \) with the \( x \) and \( y \) axes.
(c) Use limits to determine any horizontal asymptotes of \( f \).
(d) Use limits to determine any vertical asymptotes of \( f \).
(e) For each interval in the table below, indicate whether \( f \) is increasing or decreasing.

<table>
<thead>
<tr>
<th>interval</th>
<th>((-\infty, 0))</th>
<th>((0, \infty))</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(f) Determine the \( x \) coordinates of any local maximum or minimum values of \( f \).

(g) For each interval in the table below, indicate whether \( f \) is concave up or concave down.

<table>
<thead>
<tr>
<th>interval</th>
<th>((-\infty, -1/2))</th>
<th>((-1/2, 0))</th>
<th>((0, \infty))</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(h) Determine the \( x \) coordinates of any inflection points on the graph of \( f \).
7. (i) Which of the following best represents the graph of $y = f(x)$? Circle only one answer.

13. A water trough is to be made from a long strip of tin 6 ft wide by bending up at an angle $\theta$ a 2 ft strip at each side. What angle $\theta$ would maximize the cross sectional area, and thus the volume, of the trough?
9. A particle moves along a line with a position function \( s(t) \), where \( s \) is measured in meters and \( t \) in seconds. Four graphs are shown below: one corresponds to the function \( s(t) \), one to the velocity \( v(t) \) of the particle, one to its acceleration \( a(t) \) and one is unrelated.

![Graphs of s(t), v(t), and a(t)]

[3] (a) Identify the graphs of \( s(t) \), \( v(t) \) and \( a(t) \) by writing the appropriate letter (A, B, C, D) in the space provided next to the function name. (The position function \( s \) is already labeled.)

\[
\begin{align*}
\text{s} &= \phantom{D} \\
\text{v} &= \\
\text{a} &= 
\end{align*}
\]

[4] (b) Find all time intervals when the particle is slowing down, and when it is speeding up. Justify your answer.

[1] (c) Find the total distance travelled by the particle over the interval \( 3 \leq t \leq 9 \).
1. Find the derivative of each function below. Do not simplify.
   
   (a) \( f(x) = \frac{\sin(5x)}{1 + x^2} \)
   
   (b) \( g(x) = \ln \left( e^{x^2} + \sqrt{1 + x^4} \right) \)

2. Find an equation for the line that is tangent to this curve at the point where \( x = 1 \):
   
   \[ y = \ln \left( \frac{2x - 1}{2x + 1} \right). \]

3. Find an equation for the line tangent to this curve at the point \((2, 1)\):
   
   \[ x^2y^3 + x^3 - y^2 = 11. \]

5. Find each limit below or show that it does not exist. Justify your results with algebra, not with your calculator!
   
   (a) \[ \lim_{x \to 0} \left( \frac{1}{2 + x} - \frac{1}{2} \right) \]
   
   (b) \[ \lim_{x \to \infty} \left( \sqrt{x^2 + cx} - x \right) \], where \( c \) is a constant. (Answer in terms of \( c \)).

11. Residents of island Q need a new fibre-optic cable for their network. The island is 3 km offshore from the nearest point P on a straight coastline, and the nearest broadband signal source is in town T, 12 km along the shore from P. See the sketch. Underwater cable costs twice as much as dry-land cable, so the islanders decide to save money by running underwater cable from Q to R and dry-land cable from R to T. What location for point R gives the lowest cost?

![Diagram of Q, P, R, and T points with distances marked]

12. A particle moves along a vertical line, starting at time \( t = 0 \) and finishing at time \( t = 3 \). Its height at time \( t \) is
   
   \[ y = 4t^3 - 24t^2 + 21t, \quad 0 \leq t \leq 3. \]

   Find the highest and lowest points reached by this particle, and find when its speed is greatest. Give full reasons for your conclusions.