

unit 1

Number
theory

1-1 Divisibility Rules

Divisible by:	Rule
2	The number is even (it ends in 0, 2, 4, 6 or 8)
3	The sum of its digits is divisible by 3 (eg 741: $7 + 4 + 1 = 12$)
4	The last two digits are divisible by 4 (eg 932: 32 is divisible by 4)
5	The number ends in 5 or 0
6	The number is divisible by 2 and 3
8	The last three digits are divisible by 8 (eg 5256: 256 is div by 8)
9	The sum of its digits is divisible by 9 (eg 738: $7 + 3 + 8 = 18$)
10	The number ends in 0

1. Using the divisibility rules, tell whether the following numbers are divisible by each of 2, 3, 4, 5, 6, 8, 9, or 10:

a) 9840

b) 13579

c) 489458295

d) 85318075248

2. The number 79732@, where @ represents a digit from 0 - 9, is divisible by 6. What digit(s) could @ represent?

3. The number 479@213, where @ represents a digit from 0 - 9, is divisible by 9. What digit(s) could @ represent?

4. The number 7409258094@, where @ represents a digit from 0 - 9, is divisible by 2 and 9. What digit(s) could @ represent?

5. Find the smallest number greater than 200, which is divisible by 5, 6, 8 and 9.

6. True or False: (If False, give an example showing it is false)

a) If a number is divisible by 3, then it is also divisible by 9. _____

b) If a number is divisible by 9, then it is also divisible by 3. _____

c) If a number is divisible by 2 and 5, then it ends in a zero. _____

d) If a number is divisible by 4 then it must be even. _____

e) If a number is divisible by 4, then it must be divisible by 6. _____

f) If a number is divisible by 2 and 4, then it must be divisible by 8. _____

g) If you write down 4 consecutive numbers, one of them must be divisible by 4.

1-2 Factors and Multiples

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1. List all the factors of each of the following. (Are you confident you've found them all?)

a) 15

b) 28

c) 42

d) 78

e) 63

f) 120

2. List all the factors of 60 and 96. Then list the common factors.

3. List the first ten multiples of each of the following:

a) 2

b) 3

c) 6

d) 9

e) 12

f) 18

4. List all the common multiples of 12 and 18 found from the lists in last question.

5. Find the largest multiple of 7 less than (a) 100, (b) 200, (c) 500, and (d) 1000. [Calc]

6. A certain number of students went on a field trip. They could be split equally into groups of 7, or groups of 6, but if they were split into groups of 5, then one group was a student short. What is the minimum number of students that went on the trip?

1-3 Prime and Composite Numbers

Handwriting practice lines consisting of multiple rows of dashed midlines and solid top/bottom lines for letter formation.

1-3 Prime and Composite Numbers

How do we know if a number is prime?

A man called Eratosthenes (pronounced air-uh-TOS-thuh-neeZ) found out twenty-two centuries ago. He was a Greek mathematician, astronomer, athlete, poet, and librarian at the University of Alexandria, Egypt around 240 B.C.E.

Eratosthenes' method to tell if a number is prime was to imagine that a giant kitchen sieve contains all the whole numbers. When the sieve is shaken the first time the first prime number "2" stays in the sieve and all the multiples of 2 (4, 6, 8, ...) fall out through the holes in the sieve. The sieve is shaken again. This time the second prime number "3" also stays inside the sieve and the multiples of 3 (9, 15, 21, ...) fall through the sieve. The third time the sieve is shaken, the third prime number "5" is held back and lets through all the multiples of 5 that have not already fallen out. Each time the sieve is shaken the next prime number is held back and its multiples fall through the mesh. The only numbers left in the sieve are the prime numbers.

Apply this method to find all the prime numbers up to 100. When you are done, circle or highlight the prime numbers.

	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

1. Is the number one prime, composite or neither?
2. If a number is prime, can it also be composite?

3. State whether each of the following statements is true or false. If it is false give a counter-example.

Eg. The statement "All odd numbers are prime numbers" is false. A counter-example is 9.

a) All prime numbers are odd.

b) If a natural number can be expressed as a product of exactly two factors, then it is prime.

c) If a natural number is divisible by 1 and itself, then the number is prime.

d) All composite numbers can be expressed as a product of prime numbers.

e) If a natural number is not a composite number then it is prime.

f) If a composite number is divisible by a second composite number, then the first composite number is divisible by each of the prime factors of the second composite number.

4. Tell whether the following statements are Always True, Sometimes True, or Never True. Give evidence for your decision.

a) The sum of two prime numbers is prime.

b) The product of two prime numbers is prime.

c) The sum of two composite numbers is composite.

d) The product of two composite numbers is composite.

5. Do you think there are more primes between 1 and 100 or between 101 and 200? Explain your choice.

6. For each of the following, make a conjecture (prediction) then test it out with some examples. Once you're confident your conjecture is true then try to explain/prove why it is should be true.

a) Odd + Odd = _____

b) Even + Even = _____

c) Odd + Even = _____

d) Odd x Odd = _____

e) Even x Even = _____

f) Odd x Even = _____

1-4 Prime Factorization

Handwriting practice lines consisting of multiple sets of horizontal dashed lines on a solid background, designed for tracing and letter formation.

1-4 Prime Factorization

Recall that $2 \times 2 \times 2 \times 7 \times 7$ can be written in a more compact way as $2^3 \times 7^2$ or $(2^3)(7^2)$

The Prime Numbers from 1 to 50:

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47

1. Prime Factorize the following numbers. Show your factor tree for d) through l)

a) $10 =$ _____ b) $14 =$ _____ c) $31 =$ _____

d) $36 =$ _____ e) $80 =$ _____ f) $48 =$ _____

g) $72 =$ _____ h) $256 =$ _____ i) $124 =$ _____

j) $105 =$ _____ k) $429 =$ _____ l) $455 =$ _____

m) $1092 =$ _____ n) $252 =$ _____ o) $1296 =$ _____

2. Prime Factorization is sometimes called the DNA or the fingerprint of a number.
Why do you think this might be?

1-5 GCF and LCM

Note: This section will not be taught or tested, but you are free to work through it if you want.

I. Greatest Common Factor

1. Find the *GCF* of the following numbers:

a) 10, 16

b) 12, 18

c) 15, 60

d) 20, 21

e) 18, 42

f) 21, 56

g) 72, 96

h) 72, 132

i) 144, 360

j) 24, 36, 40

k) $2^3 \times 3^4 \times 5 \times 7^2$, $2^2 \times 3^2 \times 5^3 \times 7 \times 11$
(do not multiply out *GCF* or the numbers)

2. The *GCF* of two numbers is 12, and the sum of the numbers is 72. Find the two numbers.

3. Suppose that $GCF(48, A) = 6$. Find at least 3 possible values of A . Justify your answer.

4. Suppose $GCF(48, A) = 5$. What are the possible value(s) of A , if any? Justify your answer.

5. Will there always be a *GCF* of two numbers? Explain.

II. Lowest Common Multiple

(Note: Some of these numbers are repeats from the *GCF* section. Use that to your advantage.)

1. Find the LCM of the following numbers:

a) 5, 20

b) 14, 25

c) 12, 18

d) 10, 16

e) 18, 42

f) 72, 108

g) 240, 450

h) 8, 16, 24

i) 18, 20, 32

j) $2^3 \times 3^4 \times 5 \times 7^2$, $2^2 \times 3^2 \times 5^3 \times 7 \times 11$ (do not multiply out LCM or the numbers)

2. Suppose $\text{LCM}(6, A) = 36$. What are the possible value(s) of A , if any? Justify your answer

3. Suppose $\text{LCM}(18, A) = 72$. Find 3 possible values of A . Justify your answer.

4. If you just multiply two numbers together, you will always get a common multiple, but not necessarily the lowest. What kind of numbers would give you the LCM when you multiply them together? Give some examples.

5. Look at questions 1 and 6, you will notice you worked with 3 pairs of numbers that were the same in both question (10, 16), (12, 18), and (18, 42). For each pair of numbers find the product of their *GCF* and *LCM*, and then find the product of the two numbers. What do you notice? Make a conjecture (hypothesis).

(10, 16): *GCF* _____ \times *LCM* _____ = _____ $10 \times 16 =$ _____

(12, 18): *GCF* _____ \times *LCM* _____ = _____ $12 \times 18 =$ _____

(18, 42): *GCF* _____ \times *LCM* _____ = _____ $18 \times 42 =$ _____

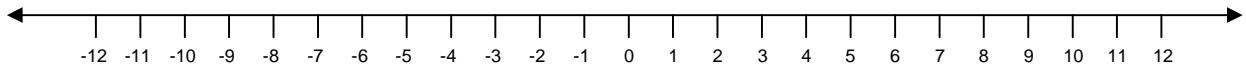
Conjecture:

Try to explain why this happens:

1.6 Integers: Addition and Subtraction

Handwriting practice lines consisting of multiple sets of horizontal dashed lines on a solid background, designed for tracing and letter formation.

1.6 Integers: Addition and Subtraction



I. Integer Addition

1. $7 + (-2) =$ _____

9. $29 + (-33) =$ _____

2. $(-8) + 16 =$ _____

10. $17 + (-6) =$ _____

3. $20 + (-13) =$ _____

11. $-3 + (-2) =$ _____

4. $-8 + (-2) =$ _____

12. $(-4) + 6 =$ _____

5. $12 + (-18) =$ _____

13. $-5 + (-5) =$ _____

6. $(-7) + (-7) =$ _____

14. $(-12) + (-18) =$ _____

7. $(-7) + 2 =$ _____

15. $-4 + \underline{\hspace{1cm}} = 8$

8. $0 + (-36) =$ _____

16. $\underline{\hspace{1cm}} + (-3) = -9$

For each of these statements, tell whether the answer will be definitely **positive**, **negative** or **dependent** (on the size of the numbers). Give 2 examples to support your answer for each question.

17. A positive plus a positive is _____:

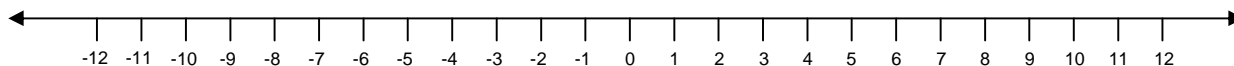
18. A positive plus a negative is _____:

19. A negative plus a positive is _____:

20. A negative plus a negative is _____:

21. Explain what you are thinking to do a question such as $-8 + 3$:

II. Integer Subtraction



1. $7 - (-2) = \underline{\hspace{2cm}}$

10. $-5 - 5 = \underline{\hspace{2cm}}$

2. $-8 - 16 = \underline{\hspace{2cm}}$

11. $17 - (-6) = \underline{\hspace{2cm}}$

3. $20 - 13 = \underline{\hspace{2cm}}$

12. $-3 - (-2) = \underline{\hspace{2cm}}$

4. $-8 - (-2) = \underline{\hspace{2cm}}$

13. $-4 - 6 = \underline{\hspace{2cm}}$

5. $12 - 18 = \underline{\hspace{2cm}}$

14. $(-12) - (-18) = \underline{\hspace{2cm}}$

6. $(-7) - (-7) = \underline{\hspace{2cm}}$

15. $-3 - \underline{\hspace{1cm}} = -9$

7. $-2 - 7 = \underline{\hspace{2cm}}$

16. $6 - \underline{\hspace{1cm}} = 13$

8. $0 - (-36) = \underline{\hspace{2cm}}$

17. $9 - \underline{\hspace{1cm}} = -2$

9. $29 - 33 = \underline{\hspace{2cm}}$

18. $\underline{\hspace{1cm}} - (-4) = 19$

For each of these statements, tell whether the answer will be definitely **positive**, **negative** or **dependent** (on the size of the numbers). Give 2 examples to support your answer for each question.

19. A positive minus a positive is _____:

20. A positive minus a negative is _____:

21. A negative minus a positive is _____:

22. A negative minus a negative is _____:

23. Explain what you are thinking to do a question such as $-8 - 3$:

24. Explain what you are thinking to do a question such as $-8 - (-3)$:

1–7 Integers: Multiplication and Division

1-7 Integers: Multiplication and Division

1. $7(8) = \underline{\hspace{2cm}}$

2. $-9(6) = \underline{\hspace{2cm}}$

3. $(-5)(12) = \underline{\hspace{2cm}}$

4. $(-4)(-8) = \underline{\hspace{2cm}}$

5. $-15 \times -3 = \underline{\hspace{2cm}}$

6. $4(-17) = \underline{\hspace{2cm}}$

7. $8(13) = \underline{\hspace{2cm}}$

8. $-12(7) = \underline{\hspace{2cm}}$

9. $20(-15) = \underline{\hspace{2cm}}$

10. $(-4)(-15) = \underline{\hspace{2cm}}$

11. $63 \div (-9) = \underline{\hspace{2cm}}$

12. $51 \div (-17) = \underline{\hspace{2cm}}$

13. $-48 \div (-6) = \underline{\hspace{2cm}}$

14. $-32 \div 2 = \underline{\hspace{2cm}}$

15. $-64 \div (8) = \underline{\hspace{2cm}}$

16. $\frac{-75}{-5} = \underline{\hspace{2cm}}$

17. $\frac{-400}{25} = \underline{\hspace{2cm}}$

18. $-6 \div \underline{\hspace{1cm}} = 1$

19. $-36 \div \underline{\hspace{1cm}} = 9$

20. $6 \times \underline{\hspace{1cm}} = -30$

21. $-12 \times \underline{\hspace{1cm}} = -108$

22. $-48 \div \underline{\hspace{1cm}} = -4$

23. $\underline{\hspace{1cm}} \div (-6) = 7$

24. $\underline{\hspace{1cm}} \times (-8) = 72$

25. Give a general rule about multiplying 2, 3, 4, or many negative numbers together.

Order of Operations

B: Simplify expressions inside brackets first.

E: Simplify expressions with exponents next (coming in unit 5).

D: Do division and multiplication next, in the order they appear going
M: from left to right.

A: Do addition and subtraction last, in the order they appear going
S: from left to right.

1. $-4 + (-8) + 15 =$ _____

2. $(-5) + (-12) + 23 =$ _____

3. $8 + (-6) + 1 + (-7) =$ _____

4. $8 - 6 - 1 - 7 =$ _____

5. $-7 - 24 - 5 =$ _____

6. $-4 - (-8) - 15 =$ _____

7. $8 - (2 - 7) =$ _____

8. $-5 - (-12) - 23 =$ _____

9. $-6 - (3 - 7) =$ _____

10. $-(11 - 4) =$ _____

11. $(-6)(4)(3) = \underline{\hspace{2cm}}$

12. $(-2)(4)(-8) = \underline{\hspace{2cm}}$

13. $(-4)(-2)(-1)(-5) = \underline{\hspace{2cm}}$

14. $\frac{(-4)(-6)(10)}{5(-3)} = \underline{\hspace{2cm}}$

15. $\frac{-8(5-8)}{2-8} = \underline{\hspace{2cm}}$

16. $4 + (-3-4)(-2) = \underline{\hspace{2cm}}$

17. $8 - 4(-2) \div 2 = \underline{\hspace{2cm}}$

18. $-3 \times 12 \div (-4) = \underline{\hspace{2cm}}$

19. $-3(2-7) + 4(-2+5) = \underline{\hspace{2cm}}$

20. $15 + 12 \div (-3) + 1 - 2(-4) = \underline{\hspace{2cm}}$

21. $-3(5 \times 2 - 7) + 4(-3 + 6 \div 3) = \underline{\hspace{2cm}}$

22. $-3(5 \times (2-7)) + 4((-3+6) \div 3) = \underline{\hspace{2cm}}$

1-8 Zero and One

Handwriting practice lines consisting of multiple rows of dashed midlines and solid top/bottom lines for letter formation.

1-8 Zero and One

Properties of Zero:

I) $n + 0 = n$

II) $n - 0 = n$

III) $n - n = 0$

IV) $n \times 0 = 0$

V) $0 \div n = 0 \ (n \neq 0)$

VI) $n \div 0$ is undefined.

where n can be any real number

NOTE: It is mathematically impossible to divide by zero!

Properties of One:

I) $n \times 1 = n$

II) $n \div 1 = n$

III) $n \div n = 1 \ (n \neq 0)$

IV) $n^1 = n$

V) $1^n = 1$

where n can be any real number.

Evaluate:

1) $a \times 0 = \underline{\hspace{2cm}}$

2) $5 \times 0 = \underline{\hspace{2cm}}$

3) $0 + 0 = \underline{\hspace{2cm}}$

4) $0 \times 5 = \underline{\hspace{2cm}}$

5) $16 \div 16 = \underline{\hspace{2cm}}$

6) $x - x = \underline{\hspace{2cm}}$

7) $0 + 1 = \underline{\hspace{2cm}}$

8) $0 \times 1 = \underline{\hspace{2cm}}$

9) $6 + 0 = \underline{\hspace{2cm}}$

10) $6 \div 6 = \underline{\hspace{2cm}}$

11) $6 \times 1 = \underline{\hspace{2cm}}$

12) $6 \div 1 = \underline{\hspace{2cm}}$

13) $\frac{49}{49} = \underline{\hspace{2cm}}$

14) $49^1 = \underline{\hspace{2cm}}$

15) $49 \div 1 = \underline{\hspace{2cm}}$

16) $49 \div 0 = \underline{\hspace{2cm}}$

17) $r \times 1 = \underline{\hspace{2cm}}$

18) $\frac{r}{r} = \underline{\hspace{2cm}} \ (r \neq 0)$

19) $0 \times r = \underline{\hspace{2cm}}$

20) $r^1 = \underline{\hspace{2cm}}$

21) $0 + b = \underline{\hspace{2cm}}$

22) $y \div 1 = \underline{\hspace{2cm}}$

23) $1 \times c = \underline{\hspace{2cm}}$

24) $f - 0 = \underline{\hspace{2cm}}$

25) $n - 0 = \underline{\hspace{2cm}}$ 26) $p - p = \underline{\hspace{2cm}}$ 27) $0 \div 8 = \underline{\hspace{2cm}}$ 28) $d \div 0 = \underline{\hspace{2cm}}$

29) $66 - 66 = \underline{\hspace{2cm}}$ 30) $0 \div g = \underline{\hspace{2cm}}$ ($g \neq 0$) 31) $24 \div 24 = \underline{\hspace{2cm}}$

32) $0 + 36 = \underline{\hspace{2cm}}$ 33) $1 \times 0 = \underline{\hspace{2cm}}$ 34) $(-4)(1) = \underline{\hspace{2cm}}$

35) $(-4)^1 = \underline{\hspace{2cm}}$ 36) $-4 \times 0 = \underline{\hspace{2cm}}$ 37) $0 \div (-4) = \underline{\hspace{2cm}}$

38) $(-4) \div 0 = \underline{\hspace{2cm}}$ 39) $0 + (-4) = \underline{\hspace{2cm}}$ 40) $12^1 \div 1^{12} = \underline{\hspace{2cm}}$

41. Explain why you can't divide by zero.

42. Is zero positive or negative?