UNIT 5

EXPONENTS
&
SQUARE ROOTS

5.1 Exponents and Bases

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		Power									
		1	2	3	4	5	6	7	8	9	10
Base	2										
	3										
	4										
	5										
	6										
	7										
	8										
	9										
	10										

I. Write as repeated multiplication:

1. 2^7 ______________________________

2. 10^5 ______________________________

3. $(-3)^4$ ______________________________

4. -3^4 ______________________________

5. $\left(\frac{1}{2}\right)^3$ ____

II. Write in exponential form:

1. eight cubed: __________ 2. The sixth power of 2: ____________

3. (-2)(-2)(-2)(-2)(-2): _________ 4. – (10)(10)(10)(10): ______________

III. Evaluate:

1. $7^2 =$____ 2. $12^2 =$____ 3. $3^4 =$____ 4. $4^3 =$____

5. $\left(\frac{2}{5}\right)^2 =$____ 6. $\left(1\frac{3}{4}\right)^2 =$____ 7. $\left(\frac{1}{2}\right)^3 =$____ 8. $\left(-\frac{1}{9}\right)^2 =$_____

9. $(0.8)^2 =$____ 10. $(1.2)^2 =$____ 11. $(0.13)^2 =$_____ 12. $(-2)^4 =$____

13. $-2^4 =$____ 14. $(-2)^3 =$____ 15. $-2^3 =$____ 16. $(-1)^7 =$____

17. $(-1)^{2020} =$___

18. $2^3 \times 3^2 =$ _____

19. $2^3 + 2^5 =$ ______

20. $5^2 - 2^5 =$ ______

21. $4^2 \div 2^4 =$ _____

22. $2^4 - 2^2 =$ _____

23. $(-3 \times 4)^2 =$______

24. $3(4)^2 =$ __________

25. $-3(4)^2 =$ ______

26. $(3 \cdot 4)^2 =$ _____

27. $-(-3 \cdot 4)^2 =$ _____

28. $2(3-4)^2 =$ ____

29. $-1^2 - 1^3 =$ _________

30. $-(3^2 - 3)^2 =$ ______

IV. Show the substitution for *m* = 4 and *n* = –3, then evaluate:

1. m^2
2. $3n^2$
3. $(3m)^2$
4. $-m^3$
5. $(n - m)^3$
6. $m^3 - n^3$

V. Problems and Thought-Provokers

1. 1 + 3 =
   1 + 3 + 5 =
   1 + 3 + 5 + 7 =
   1 + 3 + 5 + 7 + 9 =
   1 + 3 + 5 + 7 + 9 + 11 =

   What is the pattern to these sums of consecutive odd numbers?

Make a conjecture about the sums of consecutive odd numbers:

Explain why this happens. Be creative

2. The same number is subtracted from each of 71 and 58 resulting in two perfect squares. What is the number?

3. If $2^x + 3^y = 43$, where x and y are integers, then the value of $(x + y)$ is:

4. Each of the numbers 1, 2, 3, 4 is substituted, in some order, for p, q, r and s. The greatest possible value for $p^q + r^s$ is:

6. In the following equations, the letters a, b, and c represent different numbers.

$a^3 = 1 + 7$,

$3^3 = 1 + 7 + b$,

$4^3 = 1 + 7 + c$

Find the numerical value of a + b + c:

7. When $\left(5^3\right)\left(7^{52}\right)$ is evaluated, the unit's digit is:

***VI. Just for fun… You may use a calculator for the following questions if you wish.**

1. $2^5 \cdot 9^2 =$ ________________

2. $3^3 + 4^4 + 3^3 + 5^5 =$ __________________

3. $6 + 6 + 6 + (6 \times 6 \times 6) + (6 \times 6 \times 6) + (6 \times 6 \times 6) =$ ______________________

4. $(3 + 4 + 0 + 1 + 2 + 2 + 2 + 4)^6 =$ _______________

5. $12^2 + 33^2 =$ ___________

6. What is the smallest integer greater than 1 that is both a perfect square and a perfect cube?

What is the next smallest number to have the same property?

7. $15 \times 15 =$ ______ $25 \times 25 =$ ______ $35 \times 35 =$ ______

$45 \times 45 =$ ______ $55 \times 55 =$ ______ $65 \times 65 =$ ______

Can you find a pattern in the squares of these types of numbers? See if it works for additional examples.

5.2 Exponent Laws

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I. Write the following as a single power. Do not evaluate!

1. $(9^3)(9^5) =$ ____

2. $7^6 \cdot 7^6 =$ ____

3. $(4^{13})(4)(4^5) =$ ____

4. $(x^7)(x^{11}) =$ ____

5. $w(w^3)(w^{50}) =$ ____

6. $(-6)^8(-6) =$ ____

7. $24^8 \div 24^3 =$ ____

8. $\frac{3^{13}}{3^8} =$ ____

9. $t^{23} \div t =$ ______

10. $5^9 \div 5^9 =$ ____

11. $2^5 \div 2^2 =$ _____

12. $\frac{(8^{14})(8^5)}{8^{11}} =$ _______

13. $\frac{11^4(11^3 \cdot 11^7)}{11^8(11)} =$ _____

II. The Power Zero

1. $(56)^0 =$ ________
2. $(-11)^0 =$ ________
3. $-11^0 =$ ________
4. $\left(-\frac{25}{4}\right)^0 =$ ________
5. $-8(92mn)^0 =$ ________
6. $3^0 =$ ________
7. $4(3)^0 =$ ________
8. $0^3 - 3^0 =$ ________

III. Power of a Power

1. $\left(2^3\right)^2 =$ _____
2. $\left(2^2\right)^3 =$ _____
3. $\left(7^3\right)^2 =$ _____
4. $[(-3)^7]^8 =$ ________
5. $\left(12^7 \times 12^4\right)^3 =$ _____
6. $\left(\frac{2^5}{2^3}\right)^3 =$ _____
7. $\left(n^3\right)^4 =$ _____
8. $\left(\frac{n^6}{n^2}\right)^3 =$ _____
9. $\left(-x^3\right)^6 =$ ________

10. $(abc)^3 =$ ________

11. $(3m^2)^4 =$ ________

12. $(2xy)^5 =$ ________

13. $\left(\frac{3^5 \cdot 3^{12}}{3^2}\right)^3 =$ ________

14. Would it be correct to use the exponent laws on the following questions? If so, use them to write the answer as a single power. If not, evaluate the answer.

a) $3^2 \times 3^3 =$ _____

b) $2^4 + 2^5 =$ _____

c) $3^4 - 3^3 =$ _____

d) $2^8 \div 2^5 =$ _____

e) $\left(2^2 + 2^1\right)^3 =$ _____

f) $\left(2^6 \times 2^3\right)^4 =$ _____

5-3 Square Roots

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I. Evaluate the following. Estimate only when necessary:

1. $\sqrt{49} =$ ____ 2. $\sqrt{144} =$ ____ 3. $\sqrt{81} =$ ____ 4. $\sqrt{0.25} =$ ___

5. $\sqrt{0.09} =$ ____ 6. $\sqrt{12100} =$ ______ 7. $\sqrt{490000} =$ ________

8. $\sqrt{0.0144} =$

9. Why is $\sqrt{0.00144}$ different from #8?

10. $\sqrt{0.0121} =$ ________ 11. $\sqrt{\frac{4}{9}} =$ ____ 12. $\sqrt{5\frac{4}{9}} =$ ____

13. $\sqrt{\frac{50}{72}} =$ ____ 14. $\sqrt{30} =$ ____ 15. $\sqrt{78} =$ ____

16. $\sqrt{649^2} =$ ____

17. $\dfrac{\sqrt{64}}{16} =$ ____

18. $\sqrt{13^2 - 5^2} =$ ____

19. $\sqrt{13^2} - \sqrt{5^2} =$ ______

20. $\sqrt{-36} =$ ____

21. $-\sqrt{64} =$ ____

22. $2\sqrt{3^2 + 4^2} =$ ____

23. $\sqrt{3(-2)^2 - (-4)} =$ ____

24. $\sqrt{81} + \sqrt{52} =$ ___

25. $\left(\sqrt{36}\right)\left(\sqrt{4}\right) =$ ____

26. $\sqrt{36} + \sqrt{4} =$ ____

27. $\sqrt{9}\cdot\sqrt{9} =$ _____

28. $\sqrt{20}\cdot\sqrt{20} =$ ____

29. $\sqrt{\left(2^{-4}+3^{-2}\right)^{0}} =$ _____

30. $\sqrt{16+9=}$ _____

31. $\sqrt{16-9=}$ _____

32. $\sqrt{16\times 9=}$ _____

33. $\sqrt{16\div 9=}$ _____

34. $\sqrt{16}+\sqrt{9} =$ _____

35. $\sqrt{16}-\sqrt{9} =$ _____

36. $\sqrt{16}\times\sqrt{9} =$ _____

37. $\sqrt{16}\div\sqrt{9} =$ _____

II. Problems and Thought Provokers

1. What number(s) do not have a square root? Explain why not.

2. What numbers have a whole number as their square root? List all of these numbers up to 150. You should be mentally familiar with these numbers as they occur very frequently in math.

3. A square has an area of 64 cm^2, what is its perimeter?

4. Four students were trying to figure out the square-root of 42. Hewey said "the square-root of 36 is 6, and the square-root of 49 is 7, so the square root of 42 is most likely 6.5." Dewey said "when I punch it into the calculator, its says 6.407406." Louie said "I used a graphing calculator and the square root of 42 is 6.4070698." Fred said "the square root of 42 is $\sqrt{42}$." Which student gave the most accurate answer? Which student gave the best answer? What would answer would you give? Why?

5. Are there any numbers whose square root is the same as the number itself? Which number(s) are they?

6. For each statement below, tell whether it is Always, Sometimes or Never true. Explain why or provide an example why not.

a) The square root of a positive number is larger than the number.

b) The square of an even number is also even.

c) The square root of an even number is an even number.

d) The square root of a prime number is a whole number.

*7. Think about what “square root” means. What do you think “cube root” might mean. Give an example.

*8. What do you think $\sqrt{x^{16}}$ would simplify to? How can you prove your answer is correct?

*9. This chapter is called “Exponents and Square Roots”, but really it could easily just be called “Exponents”. Why?

Hint: $\left(\sqrt{9}\right)^2 = 9 = 9^1$, thus if we think of square root as an exponent then $\left(9^?\right)^2 = 9^1$ and so by our exponent law $9^{2\times ?} = 9^1$ and thus $2\times ? = 1$ so $? =$ ____. Neat eh?

5.4 The Pythagorean Theorem

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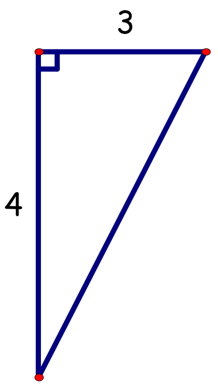
For all questions, give answer in exact form (ie. 8 or $\sqrt{45}$), then estimate the answer to one decimal place where necessary. Be sure to show the setup of Pythagoras for each question.

1. Find the length of the missing side:

a)



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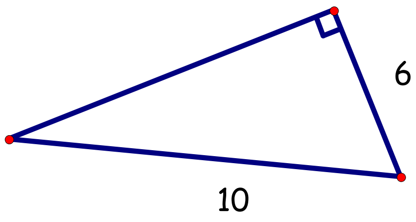

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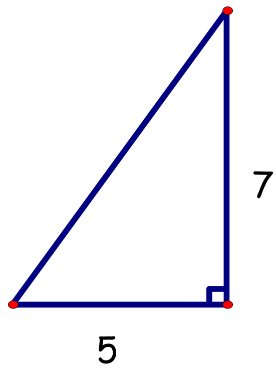
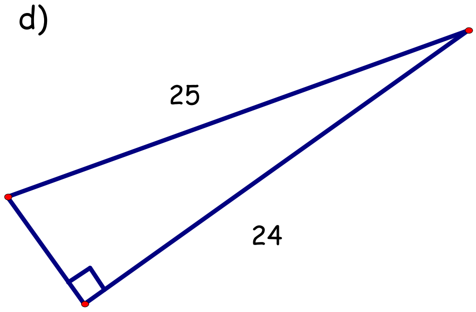
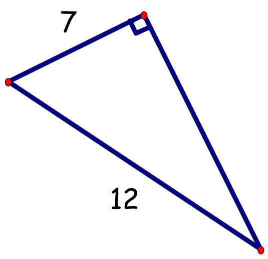


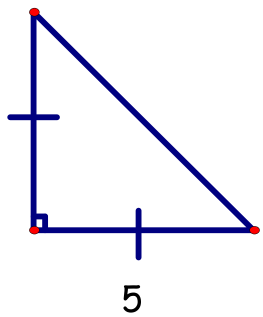
d)



e)

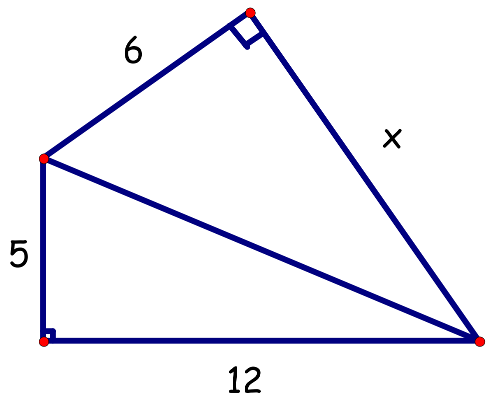


f)

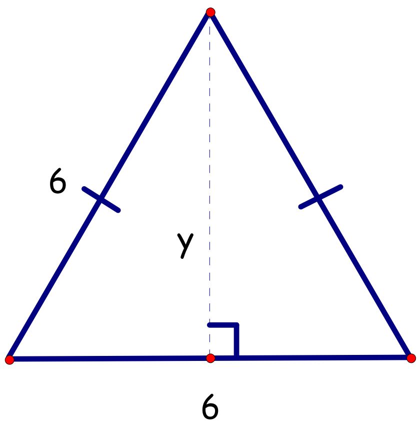


2. Solve for the indicated side (more than one-step may be required).

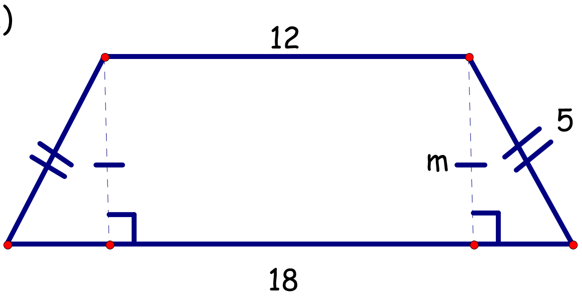
a)



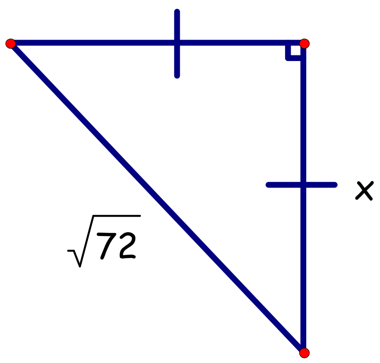
b)



c)



d)



3. Applications (Be sure to draw a diagram for each question!)

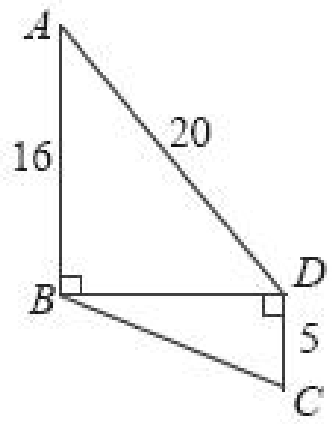
a) A rectangle has a base of 6cm and a diagonal of 10 cm, what is the height of the rectangle?

b) An 8 m ladder is placed against a wall such that the base of the ladder is 3 m from the base of the wall. How far up the wall does the ladder reach?

c) David Beckham is taking a corner kick from the south-west corner of a soccer field which measures 35m by 120m. If he wants to run to the north-east corner, how much shorter will his path there be if he runs directly there as opposed to running along the sideline (edge) of the field?

d) Two cars start at the same point and leave at the same time, the red car drives east at 30m/s, while the blue car drives north at 40m/s. How far apart will they be after 10s?

e) Juan walks from A to D to C to B and then back to A. How far has he walked?

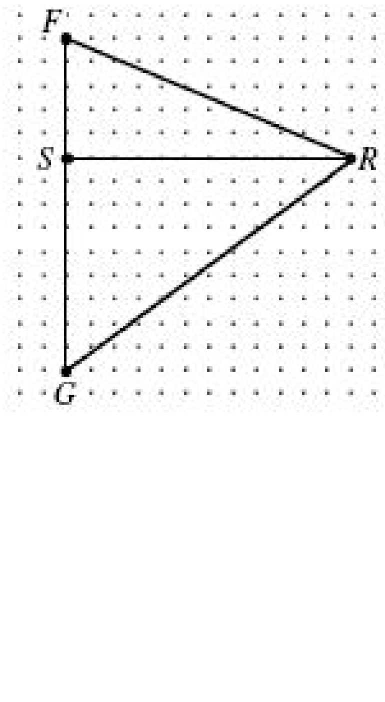


f) The homes of Fred (F), Sandy (S), Robert (R), and Guy (G) are marked on the rectangular grid with straight lines joining them as shown. FS = 5 km, SG = 9 km, and SR = 12 km. Fred is considering four routes to visit each of his friends:

i) F $\rightarrow$ R $\rightarrow$ S $\rightarrow$ G   ii) F $\rightarrow$ S $\rightarrow$ G $\rightarrow$ R

iii) F $\rightarrow$ R$\rightarrow$ G $\rightarrow$ S   iv) F $\rightarrow$ S $\rightarrow$ R $\rightarrow$ G

What is the difference between the longest and shortest trip?



4. Will the following side lengths form a right triangle? Justify your answer.

a) 5,6,7

b) 8,15,17

c) 6, 12, 18

5. Two sides of a right triangle are 5 and 7. Find all possible lengths of the third side.

6. Pythagorean Triples are sets of 3 whole number side lengths that form a right triangle. For example (3,4,5) is a Pythagorean Triple because ($3^2 + 4^2 = 5^2$).

To make Pythagorean Triples, substitute any two positive whole numbers for m and n in the following expressions: $(m > n)$

$$a = m^2 - n^2, \quad b = 2mn, \quad c = m^2 + n^2$$

eg if m = 3 and n = 1, then a = 8, b = 6, and c = 10
Find 6 more Pythagorean Triples:

1)

2)

3)

4)

5)

6)