COMBINATORICS REVIEW ANSWER KEY

1a) $(captain \;\&\; ass't.\ captain)(2\ others)$

$= ({}_{10}P_2)({}_8C_2) = (90)(28) = 2520$

b) $(boy\ captain \;\&\; girl\ ass't.\ captain\ or\ girl\ captain \;\&\; boy\ ass't.\ captain)(2\ others)$

$= 2({}_5C_1)({}_5C_1)({}_8C_2) = (2)(5)(5)(28) = 1400$

2a) ${}_{10}C_3 = 120$ b) ${}_{10}P_3 = 720$

3a) {J,Q,K,A} x 4 = 16, ${}_{16}C_5 = 4368$ b) {2 to 10} x 4 = 36, ${}_{36}C_5 = 376\,992$

c) $({}_{13}C_2)({}_{13}C_3) = (78)(286) = 22\,308$ d) 4 red cards, 1 black + 5 red cards

$({}_{26}C_4)({}_{26}C_1) + {}_{26}C_5 = (14\,950)(26) + (65\,780) = 454\,480$

e) 3-5'S, but also 2 other cards not 5's: $({}_4C_3)({}_{48}C_2) = (4)(1128) = 4512$

f) Case 1: one Queen is Q♥; Q♥, 1 other Q, 1 other ♥ and two other cards, not Q's or ♥'s:
$(1)({}_3C_1)({}_{12}C_1)({}_{36}C_2) = (3)(12)(630) = 22\,680$

Case 2: no Q♥; 2Q, 2♥, and one other card, not Q or ♥: $({}_3C_2)({}_{12}C_2)({}_{36}C_1) = (3)(66)(36) = 7\,128$

Total: 22 680 + 7 128 = 29 808

4a) $({}_{13}C_1)({}_4C_2)({}_{12}C_3)({}_4C_1)({}_4C_1)({}_4C_1) = (13)(6)(220)(4)^3 = 1\,098\,240$

b) $({}_{13}C_2)({}_4C_2)({}_4C_2)({}_{44}C_1) = (78)(6)(6)(44) = 123\,552$

c) $({}_{13}C_1)({}_4C_3)({}_{12}C_2)({}_4C_1)({}_4C_1) = (13)(4)(66)(4)^2 = 54\,912$ d) $({}_{13}C_1)({}_4C_4)({}_{48}C_1) = (13)(1)(48) = 624$

5a) ${}_{47}C_7 = 62\,891499$ b) ${}_{49}C_6 = 13\,983\,816$

c) $({}_7C_4)({}_{40}C_3) = (35)(9880) = 345\,800$; $({}_6C_4)({}_{43}C_2) = (15)(903) = 13\,545$

6a) $\frac{21!}{12!9!} = 293930$ b) $\left(\frac{11!}{7!4!}\right)\left(\frac{10!}{5!5!}\right) = (330)(252) = 83160$

7a) All possible – no boys – 1 boy = at least 2 boys

${}_{20}C_5 - {}_{12}C_5 - ({}_{12}C_4)({}_8C_1) = 15\,504 - 792 - (495)(8) = 10\,752$

b) All possible – no girls – 1 girl = at least 2 girls

${}_{20}C_5 - {}_8C_5 - ({}_8C_4)({}_{12}C_1) = 15\,504 - 56 - (70)(12) = 14\,608$

c) $({}_{12}C_5) + ({}_{12}C_4)({}_8C_1) + ({}_{12}C_3)({}_8C_2) = 792 + (495)(8) + (220)(28) = 10\,912$

8. 4x4x3 – 1 = 47

9a) 5^{15} b) $\frac{15!}{3!3!3!3!3!} = 168\,168\,000$

10. Multiples of 3 and 5: {0, 15, 30 and 45}, so there are 4 cases with first two numbers being the same; (4)(59) + (12 x 20 – 4)(58) = 13 924 or (12 x 20)(58) – 4

11a) ${}_nC_2 = 15;\ \dfrac{n!}{2(n-2)} = \dfrac{n(n-1)}{2} = 15;\ n(n-1) = 30;\ n^2 - n - 30 = 0;\ (n-6)(n+5) = 0;$

$n = 6 \text{ or } -5;$ reject $n = -5 \therefore n = 6$

b) $3{}_nC_4 = {}_nP_3\ ;\ \dfrac{3n!}{(n-4)4!} = \dfrac{n!}{(n-3)!};\ \dfrac{(n-3)!}{(n-4)!} = \dfrac{4!n!}{3n!}\ ;\ (n-3) = \dfrac{4!}{3} = 8;\ n = 11$

c) $\dfrac{4!(n-5)!}{(n-3)!} = \dfrac{4!}{(n-3)(n-4)} = 4 \quad 3! = (n-3)(n-4),\ n^2 - 7n + 6 = 0,\ n = 1 \text{ or } 6\ (\text{reject } n = 1)$

d) $x + y = 12,$ where $0 \le x \le 12$ and $0 \le y \le 12,$ and both x and y are integers;
also, $x = y = 0$ and $x = y = 1$ are solutions

12a) ${}_9P_3 = 504$ b) $({}_9C_2)(8!) = (36)(40320) = 1451520$ c) ${}_9P_6 = 60480$

13a) 7! = 5040 b) 2(6!) = 1440 c) 5040 – 1440 = 3600 d) (2)(2)(5!) = 480

e) 2(5!) = 240 f) 4!(2)(2)(2) = 192

14. ${}_nC_3 = 1\,521\,520,\ n = 210$

15a) $x^{10} + 10x^9 + 45x^8 + 120x^7 + 210x^6 + 252x^5 + 210x^4 + 120x^3 + 45x^2 + 10x + 1$

b) $19683x^9 - \dfrac{59049}{2}x^8y + \dfrac{78732}{4}x^7y^2 - \dfrac{61236}{8}x^6y^3 + \dfrac{30618}{16}x^5y^4$
$- \dfrac{10206}{32}x^4y^5 + \dfrac{2268}{64}x^3y^6 - \dfrac{324}{128}x^2y^7 + \dfrac{27}{256}xy^8 - \dfrac{y^9}{512}$

c) $59136a^6b^6$