FUNDAMENTAL COUNTING PRINCIPLE

1. How many different pasta meals can be made from 4 choices of pasta and 2 choices of sauces, if only one of each is selected for each meal?
   A. 4  B. 6  C. 8  D. 16

2. A breakfast special consists of choosing one item from each category in the following menu.
   Juice: apple, orange, grapefruit
   Toast: white, brown
   Eggs: scrambled, fried, poached
   Beverage: coffee, tea, milk
   How many different breakfast specials are possible?
   A. 11  B. 48  C. 54  D. 96

3. A restaurant offers a selection of 4 different sandwiches, 3 different soups and 4 different flavours of juice. In how many different ways can a person select one item from each category?
   A. 11  B. 18  C. 48  D. 54

4. A couple is planning an evening out. They have a choice of 4 restaurants for dinner, 6 movies following dinner, and 4 coffee establishments for after the movie. How many different ways can they plan the evening if they choose one of each?
   A. 6  B. 14  C. 48  D. 96

5. Josh wants to rent a car. He has narrowed his choices to a sedan, a compact, or an economy car. The colours available are black, red, or white. He may also choose between a standard and an automatic transmission. Determine the total number of options Josh has.

6. There are 45 multiple-choice questions on an exam with 4 possible answers for each question. How many different ways are there to complete the test?
   A. 45  B. $45 \times 4$  C. $45^4$  D. $4^{45}$

7. Katie wants to colour a rainbow. She knows the seven colours that make up the rainbow, but can't remember the correct order. How many ways could the colours be arranged assuming each colour is used only once.
   A. 28  B. 128  C. 720  D. 5040
8. Linda and Sam play a tennis match. The first person to win 2 games wins the match. In how many ways can a winner be determined?
   A. 3  B. 5  C. 6  D. 8

9. How many 6-digit numbers greater than 800 000 can be made from the digits 1, 1, 5, 5, 5 and 8?
   A. 10  B. 60  C. 64  D. 120

10. In how many ways can four colas, three iced teas, and three orange juices be distributed among ten graduates if each graduate is to receive one beverage.
    A. 36  B. 4 200  C. 604 800  D. 3 628 800

11. Each day a student chooses 1 out of 3 beverages in the school cafeteria. Over 10 days she chooses apple juice 3 times, orange juice 3 times, and lemonade 4 times. In how many different orders can this happen?
    A. 360  B. 4 200  C. 3 628 800  D. 87 091 200

12. Moving only to the right or down, how many different routes exist to get from point P to point Q?
    A. 120  
    B. 126  
    C. 180  
    D. 240

13. Moving only to the right or down, how many different routes exist to get from point A to point B?
    A. 5  
    B. 6  
    C. 7  
    D. 8
14. Numbers are formed on a calculator using seven lines which are either lit or not lit. The diagram below shows the number 8 formed using all seven lines lit. How many different symbols can be created by lighting one or more of these seven lines? (Count all the symbols, not just the ones that represent numbers.)

![Diagram of number 8](image)

15. A postal code consists of three letters and three digits arranged with a letter first, then a digit, a letter, then a digit, and a letter and a digit. If the first letter must be V, W, or X, and there are no other restrictions on the other letters or digits, determine how many different postal codes are possible. (An example of a postal code is VON 5Y2)

A. 1 259 712 B. 1 478 412 C. 1 728 000 D. 2 028 000

16. Twelve buttons differ only by colour. There are 4 red buttons, 4 green buttons and 4 yellow buttons. If the buttons are placed in a row, how many different arrangements are possible?

A. 11 880 B. 34 650 C. 19 958 400 D. 479 001 600

17. A checkerboard is an 8 × 8 game board, as shown below. Game pieces can travel only diagonally on the dark squares, one diagonal square at a time, and only in a downward direction. If a checker is placed as shown, how many possible paths are there for the checker to reach the opposite side of the game board?

![Checkerboard](image)

18. A license plate consists of 3 letters followed by 3 digits. The letters I, O, Q, U, Y and Z are not used. If repetitions of letters and digits are allowed, determine the total number of possible license plates (e.g. A B B 6 0 3).

A. 4 924 800 B. 5 832 000 C. 8 000 000 D. 17 576 000
COMBINATORICS

19. Moving only to the right or down, how many different paths exist to get from point A to point B?
   A. 22
   B. 60
   C. 120
   D. 144

20. Determine the number of different arrangements of all the letters in A P P L E P I E.
   A. 3 360
   B. 6 720
   C. 40 312
   D. 40 320

21. Determine the number of different arrangements of all the letters in the word B A L L O O N.
   A. 210
   B. 1260
   C. 2520
   D. 5040

22. Determine the number of different arrangements of all the letters in the word A P P L E S E E D.
   A. 30 240
   B. 60 480
   C. 181 440
   D. 362 880

23. Determine the number of different arrangements of the letters in the word N A N A I M O.
   A. 210
   B. 1260
   C. 2520
   D. 5040

24. If all of the letters in the word D I P L O M A are used, then how many different arrangements are possible that begin and end with an I, O, or A?
25. Moving only to the right or down, how many different paths are there from A to B?
   A. 26
   B. 52
   C. 120
   D. 252

26. The diagram shown represents a street map. If a person can only travel east or south on the streets, how many different routes are there from A to B?
   A. 60
   B. 68
   C. 80
   D. 200

27. Moving only to the right or down, how many different routes are there from A to B?
   A. 10
   B. 12
   C. 14
   D. 18

28. How many different ways are there to arrange the letters in the word T S A W W A S S E N ?
   A. 25 200
   B. 151 200
   C. 302 400
   D. 3 628 800
29. An area code is the first 3 digits in a phone number and indicates the location of either the province or the city. In Canada, the following area codes exist:

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
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<tbody>
<tr>
<td>Manitoba 204</td>
<td>Ontario 519, 613, 705, 807</td>
<td></td>
</tr>
<tr>
<td>Saskatchewan 306</td>
<td>Yukon and NW Territories 867</td>
<td></td>
</tr>
<tr>
<td>Quebec (Quebec City) 418</td>
<td>Toronto (Ontario) 289, 647</td>
<td></td>
</tr>
<tr>
<td>Montreal 514</td>
<td>Ontario (Toronto Metro) 416</td>
<td></td>
</tr>
<tr>
<td>Newfoundland 709</td>
<td>New Brunswick 506</td>
<td></td>
</tr>
<tr>
<td>Quebec 450, 819</td>
<td>Alberta (south) 403</td>
<td></td>
</tr>
<tr>
<td>British Columbia 250, 604, 778</td>
<td>Nova Scotia 902</td>
<td></td>
</tr>
<tr>
<td>Alberta 780</td>
<td></td>
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</tbody>
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Notice that there are 3 area codes for British Columbia: 250, 604 and 778. It will be necessary to add another area code as the population increases. The new area code cannot be the same as an existing code, it must begin with a 3 and end in an even number. Determine the number of possible area codes to choose from.

A. 40  B. 44  C. 49  D. 50

30. Determine the number of different arrangements of all the letters in the word TRIGONOMETRY.

A. 4989 600  B. 59 875 200  C. 119 750 400  D. 479 001 600

31. Consider the letters A, B, C, D, D, D, E, F.

a) How many different arrangements are possible using all of the letters?

b) Using all the letters, how many different arrangements are possible that start with B and end with C?

32. A word contains two M's, two E's, two N's and no other repeated letters. If one of the N's is replaced by an M, will there be greater or fewer permutations?

33. How many 5-letter arrangements are possible using the letters P, P, P, O, Y?

A. 3!  B. 5!  C. \(\frac{5!}{3!}\)  D. \(\frac{5!}{3!2!}\)
34. A hockey team has played 10 games and has a record of 5 wins, 3 losses and 2 ties. In how many ways could this have happened if after the first 4 games the team’s record was 3 wins and a loss?
   A. 90   B. 94   C. 360   D. 630

35. How many permutations are there using all of the letters in the word P E P P E R?
   A. 60   B. 120   C. 360   D. 720

36. Determine the number of pathways from point A to point B if only moves to the right and down are permitted.
   A. 18   B. 19   C. 23   D. 47

37. How many even 4-digit whole numbers are there? For example, 1220 is acceptable but 0678 is not.
   A. 3600   B. 3645   C. 4500   D. 5000

38. North American area codes are three digit numbers. Before 1995, area codes had the following restrictions: the first digit could not be 0 or 8, the second digit was either 0 or 1, and the third digit was any number from 1 through 9 inclusive. Under these rules, how many different area codes were possible?
   A. 112   B. 120   C. 144   D. 504

39. In a particular city, all of the streets run continuously north-south or east-west. The mayor lives 4 blocks east and 5 blocks north of city hall. Determine the number of different routes, 9 blocks in length, that the mayor can take to get to city hall.
   A. 20   B. 126   C. 3024   D. 15120
40. A soccer team played 12 games in a season. They won 6 games, lost 4 games, and tied 2 games. In how many different orders could this have occurred?
   A. 576  B. 13 860  C. 9 979 200  D. 31 933 440

41. There are 2 English books, 3 Chemistry books and 4 Mathematics books to be arranged on a shelf.
   a) If all the English books are identical, all the Chemistry books are identical and all the Mathematics books are identical, in how many different ways can they be arranged on the shelf?
   b) If the English books, Chemistry books and Mathematics books are all different, in how many different ways can they be arranged on the shelf?
   c) If all the English books, Chemistry books and Mathematics books are different, in how many different ways can they be arranged on the shelf if the Chemistry books have to be grouped together?
   d) If all the English books, Chemistry books and Mathematics books are different, in how many different ways can they be arranged on the shelf if all the same subject books must be grouped together?

42. Determine the number of different arrangements of all the letters in the word PARALLEL if
   a) there are no restrictions.
   b) the A's must be together.
   c) the first letter must be an A and the last letter must be an A.
   d) the first letter must be a vowel.
43. Moving only to the right or down, how many different routes are there from A to B?
   A. 19
   B. 22
   C. 24
   D. 37

44. How many odd 3-digit whole numbers are there? For example, 203 is acceptable but 023 is not.
   A. 360
   B. 450
   C. 500
   D. 900

45. Car license plates consist of 6 characters. Each of the first 3 characters can be any letter from A to Z inclusive except I or O. Each of the last 3 characters can be any digit from 2 to 9 inclusive. If repetitions of letters and digits are not allowed, how many different license plates are possible?
   An example of this format is GRT492.
   A. 4080384
   B. 5241600
   C. 7077888
   D. 11232000

46. Moving only to the right or down, determine the number of different pathways from A to B.
   A. 13
   B. 24
   C. 60
   D. 80

47. How many arrangements of all of the letters of the word R E A S O N are there if the arrangement must start with an S?
48. Determine the number of different arrangements using all the letters of the word A C C E S S E S that
    a) begin with exactly two S's.

    b) begin with at least two S's.

49. At a car dealership, the manager wants to line up 10 cars of the same model in the parking lot. There are 3 red cars, 2 blue cars, and 5 green cars. If all 10 cars are lined up in a row facing forward, determine the number of possible car arrangements if the blue cars cannot be together.

50. How many different 4-letter arrangements are possible using any 2 letters from the word SMILE and any 2 letters from the word FROG?

51. In how many more ways can 4 people be arranged in a row than if they were arranged in a circle?
    A. 1  B. 6  C. 12  D. 18

52. In how many different ways can a family of 5 people (2 parents and 3 children) sit in a row if a parent must sit at the end of each row?
    A. 6  B. 12  C. 24  D. 120

53. A volleyball team made up of 6 players stands in a line facing the camera for a picture. If Joan and Emily must be together, then how many different arrangements are possible for the picture?

54. How many different ways can 4 girls and 4 boys be arranged in a row if the girls and the boys must alternate?
55. There are 5 men and 4 women to be seated in a row. How many arrangements are possible if two men must sit at the beginning of the row and two men must sit at the end of the row?

56. Determine the number of 4-letter arrangements that can be made from the letters in the word B A L I E T.

57. There are 4 boys and 3 girls to be seated in a row.
   a) How many arrangements are possible if Alexandra, one of the 7 people, must sit in the middle?
   b) How many arrangements are possible if Shawn and Dave, 2 of the boys, cannot sit beside each other?

58. How many ways can four couples be seated in a row of 8 chairs if each person wants to sit beside their partner?
   A. 4! \cdot 2^4  
   B. 4! \cdot 2^3  
   C. 4! \cdot 2^2  
   D. 4! \cdot 2

59. The number of ways to arrange 3 boys and 3 girls in a circle if the boys and girls alternate is:
   A. 5! \cdot 2  
   B. 5!  
   C. 3! \cdot 3!  
   D. 3! \cdot 2!

60. A family of 6 (2 parents and 4 children) sit in a row at a theatre. A parent must sit at either end with the 4 children between them. In how many ways can the family be seated?
   A. 24  
   B. 48  
   C. 120  
   D. 720
61. How many ways can 2 boys and 2 girls be seated in a row if they cannot sit beside someone of the same gender?

**FACTORIALS**

62. Evaluate: \( \frac{200!}{198!} \)

A. 2  
B. 200  
C. 39 800  
D. infinity

63. Solve: \( \frac{(n-1)!}{(n-3)!} = 30 \)

64. Simplify: \( \frac {(n-2)!}{(n-1)!} \)

A. \( \frac{n-3}{n-1} \)  
B. \( n-2 \)  
C. \( \frac{1}{n-1} \)  
D. \( \frac{1}{n(n-1)} \)

65. Simplify: \( \frac{n(n+1)!}{(n-1)!} \)

A. \( 2n! \)  
B. \( n!(n^2+n) \)  
C. \( 2n \)  
D. \( n^3 + n^2 \)

66. Solve: \( \frac{n!}{(n-2)!\ 4!} = 10 \)

67. Solve: \( \frac{n!}{(n-2)!3!} = 5 \)
68. Simplify: \( \frac{6!}{3!2!} \)
   
   A. 1  B. 20  C. 60  D. 120

69. Solve: \( \frac{14!}{12!} = 14n \)

70. Simplify: \( \frac{(n - 2)!(n + 1)!}{(n!)^2} \)
   
   A. \( \frac{1}{n} \)  B. \( \frac{1}{n - 1} \)  C. \( \frac{n - 1}{n(n + 1)} \)  D. \( \frac{n + 1}{n(n - 1)} \)

71. Solve: \( \frac{n!}{4!} = \frac{(n + 1)!}{6!} \)