PERMUTATIONS AND COMBINATIONS

72. A special combination lock that has 60 numbers on the dial works by turning it first to the right, then to the left, and then to the right, with 3 different selected numbers needed to open the lock. The selection of these 3 numbers is an example of

A. a permutation
B. a combination
C. both a combination and a permutation
D. neither a combination nor a permutation

73. A student has 7 different textbooks. Which expression gives the number of different ways 4 of these books can be selected and arranged on a shelf?

A. $4!$
B. $\frac{7!}{4!}$
C. ${}_7C_4$
D. ${}_7P_4$

74. An Olympic final race has 7 competitors. In how many ways could the gold, silver and bronze medals be awarded?

A. 21
B. 35
C. 210
D. 5040

75. How many different committees of 2 people can be selected from 5 people?

A. $\frac{5!}{2!}$
B. $\frac{5!}{3!}$
C. $\frac{5!}{2!3!}$
D. $5!$

76. A bowl contains an apple, an orange, a plum and a banana. How many different pairs of fruit can be selected from the bowl?

A. ${}_4P_2$
B. ${}_2P_4$
C. ${}_4C_2$
D. ${}_2C_4$

77. Assume a car license plate consists of 7 characters. The first 3 characters can be any of the letters from A to F, but no letter can be repeated. The next 3 characters can be any of the digits from 1 to 9, but no digit can be repeated. The last character can be any of the letters X, Y or Z. An example of this format is: B F A 6 4 8 Y. How many license plates are possible?

A. 5 040
B. 181 440
C. 472 392
D. 4 084 080

78. a) Using the digits 0, 1, 3, 4, 5, and 8, how many 4-digit numbers are possible if repetition of digits is not allowed? (Note: The first digit cannot be zero.)

b) Using the digits 0, 1, 3, 4, 5, and 8, how many 4-digit numbers greater than 4000 and divisible by 5 are possible if repetition of digits is not allowed? (Note: The first digit cannot be zero.)

79. Consider the digits 0, 2, 4, 5, 6, 8. How many 3-digit even numbers less than 700 can be formed if repetition of digits is not allowed? (Note: The first digit cannot be zero.)

80. Codes with 5 digits are made from the digits 1, 2, 3, 4, 5, 6, 7, 8, 9. If repetitions are not permitted and each code must contain 2 odd digits followed by 3 even digits, determine the number of different codes that can be made.

A. 126 B. 480 C. 1600 D. 15 120

81. A toy box contains 4 different cars and 6 different trucks.

a) In how many ways can a collection of 5 toys be chosen if the collection must consist of 2 cars and 3 trucks?

b) In how many ways can a collection of 5 toys be chosen if the collection must consist of at least 3 cars?

82. A man has 7 different pets and wishes to photograph them 3 at a time arranged in a line. How many different arrangements are possible?

A. 21 B. 35 C. 210 D. 840

83. The students at a graduation dinner are separated into groups to be seated at 10 different tables. The order in which these 10 groups will approach the buffet is to be determined randomly. In how many ways can this order be determined?

A. 10^{10} B. ${}_{10}C_{10}$ C. ${}_{10}P_{10}$ D. 10×10

84. Which term is equivalent to ${}_nC_2$?

A. $n^2 - 2n$ B. $n^2 - n$ C. $\frac{1}{2}\left(n^2 - 2n\right)$ D. $\frac{1}{2}\left(n^2 - n\right)$

85. A class has 30 students.

a) How many ways can a committee of 3 people be selected from the class?

b) How many ways can an executive committee consisting of 3 people (president, vice-president, secretary) be selected from the class?

c) If there are 10 boys and 20 girls in the class, how many ways can a committee of 3 people be selected if the committee must contain 1 boy and 2 girls?

86. How many ways can 6 people be seated at a round table?

A. ${}_6C_6$ B. ${}_6C_5$ C. 5! D. 6!

87. When you play lotto 5-30 , you must choose 5 different integers from 1 to 30. How many combinations are possible?

A. $\frac{30!}{5!\ 25!}$ B. $\frac{30!}{25!}$ C. 25! D. $\frac{30!}{5!}$

The winner of a lottery can choose 4 vehicles from 12 different cars, 8 different trucks and 5 different motorcycles. Use this information for the next 2 questions.

88. How many choices of 4 vehicles are possible?

A. 480 B. 570 C. 12 650 D. 303 600

89. How many choices of 4 vehicles are possible if there must be at least one car?

A. 1 171 B. 3 432 C. 9 218 D. 11 935

90. Express ${}_{33}C_5$ using factorial notation.

A. $\frac{33!}{5!}$ B. $\frac{33!}{28!}$ C. $\frac{33!}{5!28!}$ D. 28!

91. Suppose you play a game of cards in which only three cards are dealt from a standard 52-card deck. How many ways are there to obtain one pair? (2 cards of the same rank and 1 card of a different rank.) An example of a hand that contains one pair is 2 jacks and 1 five.

A. 1 014 B. 1 872 C. 3 744 D. 3 900

92. A class of 34 students consists of 20 girls and 14 boys. How many different committees of 5 girls and 3 boys can be formed from this class?

A. 2 282 280 B. 5 643 456 C. 18 643 456 D. 40 632 288 320

93. Solve: ${}_nP_2 = 42$

A. 2 B. 6 C. 7 D. 42

94. Solve: ${}_nC_3 = {}_nP_2$

A. 6 B. 8 C. 1, 8 D. 0, 1, 8

95. There are 7 boys and 5 girls in a group of students.

    a) Calculate the number of ways that a committee of 4 students can be chosen from this group if the committee must have exactly 1 boy.

    b) If the committee of 4 students must have a female president, a male vice-president, and 2 other members chosen from the remaining students, how many ways can such a committee be chosen?

96. Suppose you play a game of cards in which only four cards are dealt from a standard 52-card deck. How many ways are there to obtain three of a kind? (3 cards of the same rank and 1 card of a different rank, for example 3 tens and 1 queen)

    A. 1 872 B. 2 496 C. 2 548 D. 2 704

97. A theatre company of 13 actors consists of 8 men and 5 women.

    a) How many different ways are there to choose a group of 7 with exactly 3 men from this company?

    b) How many different ways are there to choose a group of 6 with at least 4 women?

98. In a standard deck of 52 cards, how many different 5-card hands are there that contain at most two face cards?

    A. 652 080 B. 844 272 C. 1 748 760 D. 2 406 768

99. Solve: $\frac{{}_nC_5}{{}_nC_6} = \frac{2}{5}$

100. A coach needs to choose an 8-member volleyball team from 10 males and 12 females. If there must be at least 3 of each gender on the team, how many different teams are possible?

A. 103 950 B. 150 480 C. 254 430 D. 319 770

101. Evaluate: $\frac{{}_9P_4}{{}_9P_5}$

A. 5 B. 1 C. $\frac{4}{5}$ D. $\frac{1}{5}$

102. Evaluate: $\frac{{}_7P_2}{{}_7P_5}$

103. Evaluate: $\sum_{k=1}^{4} {}_4C_k$

A. 1 B. 4 C. 15 D. 16

104. A soccer coach must choose 3 out of 10 players to kick tie-breaking penalty shots. Assuming the coach must designate the order of the 3 players, determine the number of different arrangements she has available.

A. $\frac{10!}{7!}$ B. $\frac{10!}{3!}$ C. $\frac{10!}{3!7!}$ D. $\frac{10!}{3!3!4!}$

105. In a standard deck of 52 cards, how many different 4-card hands are there that contain at most one heart?

A. 91 403 B. 118 807 C. 188 474 D. 201 058

106. Assuming that at least one coin is used, how many different sums of money can be made from the following coins: a penny, a nickel, a dime, a quarter and a dollar?

A. 16 B. 31 C. 32 D. 120

107. There are five boys and six girls on a grad committee.

a) In how many ways can a sub-committee of two boys and two girls be selected from the committee?

b) In how many ways can a sub-committee of four people be selected if there must be at least one girl on the sub-committee?

108. When playing the 6 49 lottery, a customer must choose 6 different numbers from 1 to 49 inclusive. How many combinations are possible?

A. $49!$ B. $\frac{49!}{6!\,43!}$ C. $\frac{49!}{43!}$ D. $\frac{49!}{6!}$

109. From a class of 12 boys and 10 girls a committee of 3 people is selected. How many different committees have at least 1 boy?

A. 120 B. 540 C. 1 420 D. 1 540

110. a) How many groups of 3 chairs can be chosen from 7 chairs if the chairs are all different colours?

b) How many different ways can 7 chairs be arranged in a row if 2 of the chairs are blue, 3 are yellow, 1 is red and 1 is green? (Assume that all of the chairs are identical except for colour.)

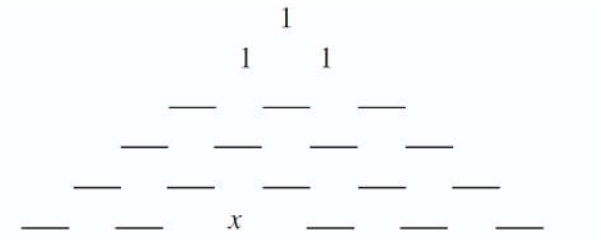
111. Given Pascal's triangle below, which of the following is equivalent to the value of x?

A. ${}_4C_2$

B. ${}_5C_2$

C. ${}_6C_2$

D. ${}_6C_3$



112. Solve algebraically using factorial notation: ${}_nP_2 = 90$

113. At a meeting, every person shakes hands with every other person exactly once. If there are 36 handshakes in total, how many people were at the meeting?

114. Henri has 83 different books in his collection. In how many ways can he select a group of 10 of these books?

A. ${}_{83}P_{10}$ B. ${}_{83}C_{10}$ C. $10!$ D. $\frac{83!}{10!}$

115. How many ways can a president, a vice-president, and a secretary be elected from a class of 22 students?

A. $(22)(21)(20)$ B. $(22!)(21!)(20!)$ C. $\frac{22!}{19!3!}$ D. $\frac{22!}{3!}$

116. Solve the following equation: ${}_nP_2 = {}_nC_3$

117. You want to select a committee of 3 girls and 2 boys from a group of 8 girls and 6 boys.

a) How many committees are possible?

b) John is one of the boys and Marie is one of the girls. Either John or Marie must be on the committee, but not both. How many committees are possible?

118. In a particular class, there are 5 girls and 10 boys.

a) How many committees of 7 people can be formed if there must be 3 girls and 4 boys on the committee?

b) How many committees of 7 people can be formed if there is at least one girl on the committee?

119. Determine the value of n. ${}_nC_9 = {}_nC_3$

120. From a standard deck of 52 cards, how many 5-card hands can be formed containing at least 4 clubs?

A. 20 592 B. 27 885 C. 29 172 D. 34 320

121. In a standard deck of 52 cards, determine the number of 5-card hands that must contain at least 3 queens.

A. 4512 B. 4560 C. 4704 D. 4752

122. A dance group has twelve people from which five need to be chosen to compete. If Bob and Nancy are required to be among the five selected, how many different five-member teams are possible?

A. 120 B. 220 C. 252 D. 792

123. Determine the equivalent expression for ${}_{22}C_8$.

A. ${}_8C_{22}$ B. ${}_{20}C_6$ C. ${}_{22}C_{14}$ D. $\frac{{}_{22}P_8}{14}$

124. Solve: ${}_{n-1}P_2 = 42$

BINOMIAL EXPANSION

125. How many terms are in the expansion of $(2x+y)^9$?

A. 8 B. 9 C. 10 D. 11

126. Determine the 10th term in the expansion of: $\left(2x-\frac{1}{y}\right)^{10}$

127. Determine the 5th term in the expansion of: $\left(x-\frac{y}{2}\right)^7$

A. $\frac{35}{8}x^4y^3$ B. $\frac{35}{16}x^3y^4$ C. $-\frac{35}{8}x^4y^3$ D. $-\frac{35}{16}x^3y^4$

128. Determine the 4th term in the expansion of: $(x-2)^6$

A. $120x^2$ B. $240x^2$ C. $-160x^3$ D. $-320x^3$

129. Determine the 4th term in the expansion of: $(x-2y)^5$

A. $-80x^2y^3$ B. $-40x^3y^2$ C. $40x^3y^2$ D. $80x^2y^4$

130. Determine the 3rd term in the expansion of: $(x-y)^{10}$

A. $-45x^8y^2$ B. $-120x^7y^3$ C. $45x^8y^2$ D. $120x^7y^3$

131. Determine the 3rd term in the expansion of: $(2x+y)^6$

A. $15x^4y^2$ B. $240x^4y^2$ C. $120x^3y^3$ D. $160x^3y^3$

132. In the expansion of $(2a-3b)^6$, determine the coefficient of the term containing a^4b^2.

A. $-4\,320$ B. 864 C. 2 160 D. 2 880

133. Determine the coefficient of the 3rd term in the expansion of $(x+2y)^7$.

A. 21 B. 35 C. 84 D. 140

134. In the expansion of $(x+y)^{10}$, determine the coefficient of the term containing x^8y^2.

A. 9 B. 10 C. 36 D. 45

135. In the expansion of $(a^2-b)^4$, determine the middle term.

A. a^2b^2 B. $6a^2b^2$ C. a^4b^2 D. $6a^4b^2$

136. If $64a^6$ is the first term in the expansion of $(2a-b)^n$, determine the coefficient of the 4th term.

A. -160 B. -20 C. 20 D. 160

137. The 10th term in the expansion of $\left(x-\frac{1}{2}\right)^n$ is $-\frac{1001}{256}x^5$. Determine n.

A. 13 B. 14 C. 15 D. 16

138. Determine the 8th term in the expansion of: $(2x-y)^{11}$

A. $-5280x^4y^7$ B. $-2640x^4y^7$ C. $1320x^3y^8$ D. $990x^3y^8$

139. Determine the first 3 terms in the expansion of: $(x-2y)^7$

140. a) In the expansion of $\left(\frac{3}{x^2}-4x^5\right)^8$, determine the 3rd term.

b) In the expansion of $\left(\frac{3}{x^2}-4x^5\right)^n$, the 6th term contains x^{25}. Solve for n.

141. The 4th term in the expansion of $\left(qx^2-\frac{3}{x}\right)^{10}$ is $414\,720x^{11}$.
Determine the value of q.

142. Determine the 6th term in the expansion of: $\left(\frac{1}{2x^2}-4x\right)^8$

143. The 4th term in the expansion of $(x-2)^n$ is $-80x^2$. Determine the value of n.

A. 4 B. 5 C. 6 D. 7

144. Determine the 5th term in the expansion of: $(3x+2y)^n$, where $n \geq 6$

A. ${}_nC_4(3x)^{n-4}(2y)^4$ B. ${}_nC_5(3x)^{n-5}(2y)^5$ C. ${}_nC_4(3x)^4(2y)^{n-4}$ D. ${}_nC_5(3x)^5(2y)^{n-5}$

145. Determine the 4th term in the expansion of: $\left(4x-\frac{y}{2}\right)^8$

A. $-7168x^5y^3$ B. $-128x^5y^3$ C. $16x^4y^4$ D. $1120x^4y^4$

146. In the binomial expansion of $\left(\frac{5}{x^3} - x^2\right)^{15}$ there is a term that, when simplified, contains x^5. State which term contains x^5 and completely simplify this term.

147. Determine the 6th term in the expansion of: $\left(3x^4 - \frac{1}{x^3}\right)^9$

148. Determine the 14th term in the expansion of: $\left(\frac{3}{2x^2} - x^3\right)^{16}$

149. Determine the 7th term in the expansion of: $\left(3a - 2b^2\right)^{10}$

150. Consider the following statements regarding the expansion of $(a+b)^4$, written in descending powers of a.

Statement 1: The total number of terms is 5.

Statement 2: The middle term is $6a^2b^2$.

Statement 3: The sum of the leading coefficients of all the terms is 14.

Statement 4: For the term $4a^3b^m$, the value of m is 1.

Statement 5: The leading coefficient of the first term is ${}_4C_1$.

Which statements are true?

151. Determine the first three terms in the expansion of $(x+2y)^{10}$.

A. $x^{10}+10x^9y+90x^8y^2$

B. $x^{10}+20x^9y+180x^8y^2$

C. $x^{10}+10x^9y+45x^8y^2$

D. $x^{10}+20x^9y+45x^8y^2$

152. Consider the geometric sequence $1,\ (a+b),\ (a+b)^2,\ \ldots$
Which term of this sequence, when expanded, contains the expression $35a^4b^3$?

A. 5th term B. 6th term C. 7th term D. 8th term

153. Which term in the expansion of $\left(\frac{1}{2x^2}-x^3\right)^{10}$ is a constant?

A. 4^{th} B. 5^{th} C. 6^{th} D. 11^{th}