AUG 1997

69. Determine the area of the rectangle formed by the horizontal and vertical tangents to the conic \( \frac{(x-1)^2}{9} + \frac{(y+2)^2}{16} = 1 \).

A. 12 square units  B. 24 square units  C. 48 square units  D. 144 square units

70. A parabolic arch supports a bridge over a canal, as shown in the diagram. If an equation of the arch is \( y = -\frac{1}{30}x^2 + 5 \), determine the width \( w \) of the canal. (Accurate to two decimal places.)

A. 10.00  
B. 12.25  
C. 17.32  
D. 24.49

71. Which of the following values for the constants \( A \) and \( B \) will cause the equation \( Ax + By = 0 \) to represent a rectangular hyperbola with vertices on the line \( y = -x \) ?

A. \( A > 0, B < 0 \)  
B. \( A > 0, B > 0 \)  
C. \( A < 0, B > 0 \)  
D. \( A = 0, B < 0 \)

72. If a function is defined by the equation \( f(t) = t^2 - 4t \), then which conic is represented by \( f(x) + 2f(y) = 0 \)?

A. circle  
B. ellipse  
C. parabola  
D. hyperbola

JAN 1998

73. Find the radius of the circle \( (x+5)^2 + (y-3)^2 = 16 \).

A. 2  
B. 4  
C. 8  
D. 16

74. What is the length of the major axis of the ellipse \( \frac{x^2}{36} + \frac{y^2}{64} = 1 \)?

A. 6  
B. 8  
C. 12  
D. 16
75. Determine an equation of the hyperbola graphed below.

A. \[ \frac{x^2}{4} - \frac{y^2}{9} = 1 \]
B. \[ \frac{x^2}{9} - \frac{y^2}{4} = 1 \]
C. \[ \frac{x^2}{4} - \frac{y^2}{9} = -1 \]
D. \[ \frac{x^2}{9} - \frac{y^2}{4} = -1 \]

76. A parabola with the vertex \((-3, 2)\) has a horizontal axis of symmetry and passes through the point \((5,4)\). Determine an equation of the parabola.

A. \[ x = \frac{1}{2}(y - 2)^2 - 3 \]  
B. \[ x = 2(y - 2)^2 - 3 \]  
C. \[ x = \frac{1}{2}(y + 2)^2 + 3 \]  
D. \[ x = 2(y + 2)^2 + 3 \]

77. A point \(P(x, y)\) moves such that the slope of the line through \(P\) and \(A(-6, 0)\) is always 3 greater than the slope of the line through \(P\) and \(B(6, 0)\). Determine an equation of this locus.

A. \[ \frac{y}{x + 6} + 3 = \frac{y}{x - 6} \]  
B. \[ \frac{y}{x + 6} = \frac{y}{x - 6} + 3 \]  
C. \[ \sqrt{(x + 6)^2 + y^2} = \sqrt{(x - 6)^2 + y^2} + 3 \]  
D. \[ \sqrt{(x + 6)^2 + y^2} + 3 = \sqrt{(x - 6)^2 + y^2} \]

78. The arch of a bridge over a river has the shape of a parabola. The base is 40 m wide and the maximum height of the arch is 10 m. Determine the height \(h\) of the arch at a distance of 12 m from the centre of the arch. (Accurate to at least one decimal place.)
CONICS

APR 1998

79. Determine the value of \( k \) so that the graph of \( x = 2y^2 + ky + 33 \) has \( y = -3 \) as its axis of symmetry.
   A. \(-12\)  B. \(-6\)  C. \(6\)  D. \(12\)

JUN 1998

80. Which conic is described by the equation \( 4x^2 + 5x - 3y + 2 = 0 \)
   A. circle  B. ellipse  C. parabola  D. hyperbola

81. Determine the length of the minor axis of the ellipse \( \frac{(x - 1)^2}{25} + \frac{(y + 3)^2}{49} = 1 \).
   A. 5  B. 7  C. 10  D. 14

82. Change \( x^2 + y^2 - 4x + 2y - 4 = 0 \) to standard form.
   A. \((x - 2)^2 + (y + 1)^2 = 1\)  B. \((x - 2)^2 + (y + 1)^2 = 4\)
   C. \((x - 2)^2 + (y + 1)^2 = 9\)  D. \((x - 2)^2 + (y + 1)^2 = 10\)

83. Which equation could be used to determine the coordinates of a point on the x-axis that is equidistant from \( A(-2, 1) \) and \( B(3, 4) \)?
   A. \(\sqrt{(x - 2)^2 + (0 + 1)^2} = \sqrt{(x + 3)^2 + (0 + 4)^2}\)  B. \(\sqrt{(x + 2)^2 + (0 - 1)^2} = \sqrt{(x - 3)^2 + (0 - 4)^2}\)
   C. \(\sqrt{(0 + 2)^2 + (y - 1)^2} = \sqrt{(0 - 3)^2 + (y - 4)^2}\)  D. \(\sqrt{(0 - 2)^2 + (y + 1)^2} = \sqrt{(0 + 3)^2 + (y + 4)^2}\)

84. Find an equation of the rectangular hyperbola with vertices \((4, -1)\) and \((4, 5)\).
   A. \((x + 4)^2 - (y + 2)^2 = -9\)  B. \((x - 4)^2 - (y - 2)^2 = -9\)
   C. \((x + 4)^2 - (y + 2)^2 = 9\)  D. \((x - 4)^2 - (y - 2)^2 = 9\)
85. The cross-section of an irrigation canal is in the shape of a semi-ellipse with a width of 8 m and a depth of 3 m. Determine the width \( w \) of the water surface when the depth is 2 m. (Accurate to 2 decimal places.)

A. 5.96 m  
B. 6.83 m  
C. 7.38 m  
D. 7.54 m

**AUG 1998**

86. What are the equations of the asymptotes for the hyperbola \( x^2 - y^2 = -7 \)?

A. \( y = \pm \sqrt{7}x \)  
B. \( x = \pm \sqrt{7}y \)  
C. \( y = \pm x \)  
D. \( y = 0, x = 0 \)

87. The length of the major axis of an ellipse is four times the length of the minor axis, and \( P(2, 4) \) is a point on the ellipse. Determine the equation of the ellipse if its centre is \((0, 0)\) and its major axis is on the \( y \)-axis.

A. \( x^2 + 16y^2 = 80 \)  
B. \( 16x^2 + y^2 = 80 \)  
C. \( x^2 + 16y^2 = 260 \)  
D. \( 16x^2 + y^2 = 260 \)

**JAN 1999**

88. Which conic is represented by the equation \( 3x^2 + 6y^2 - 12y - 4 = 0 \)?

A. circle  
B. ellipse  
C. parabola  
D. hyperbola

89. Determine an equation of the ellipse graphed below.

A. \( \frac{(x - 2)^2}{36} + \frac{(y + 1)^2}{16} = 1 \)

B. \( \frac{(x + 2)^2}{36} + \frac{(y - 1)^2}{16} = 1 \)

C. \( \frac{(x - 2)^2}{16} + \frac{(y + 1)^2}{36} = 1 \)

D. \( \frac{(x + 2)^2}{16} + \frac{(y - 1)^2}{36} = 1 \)
90. A point \((x, y)\) moves such that it is equidistant from the point \((3, 8)\) and the line \(y = 0\). Determine an equation of this locus.

A. \((x - 3)^2 + (y - 8)^2 = x^2\)  
B. \((x - 3)^2 + (y - 8)^2 = y^2\)  
C. \(y = 4\)  
D. \((x + 3)^2 + (y + 8)^2 = y^2\)

91. Change \(2x^2 + y^2 - 8x - 6y + 1 = 0\) to standard form.

A. \(\frac{(x - 2)^2}{6} + \frac{(y - 3)^2}{12} = 1\)  
B. \(\frac{(x - 2)^2}{8} + \frac{(y - 3)^2}{16} = 1\)  
C. \(\frac{(x - 4)^2}{12} + \frac{(y - 3)^2}{24} = 1\)  
D. \(\frac{(x - 4)^2}{20} + \frac{(y - 3)^2}{24} = 1\)

92. A parabolic arch of maximum height 40 m is 30 m wide at ground level. A horizontal beam of length 12 m is placed across the arch as shown in the diagram below. Calculate the height, \(h\), of the beam above the ground.

A. 14.4 m  
B. 16 m  
C. 32 m  
D. 33.6 m
93. A hyperbola has centre $(1, 4)$ and one vertex at $(1, 10)$. If the asymptotes have slopes $\pm \frac{3}{2}$, determine the equation of the hyperbola in standard form and graph the conic on the grid below.

![Graph of a hyperbola with center at (1, 4) and one vertex at (1, 10).](image)

**JUN 1999**

94. For which of the following values of the constant $A$ will the equation $Ax^2 + 6y^2 = 36$ represent an ellipse?

A. $-12$  
B. $-6$  
C. $0$  
D. $12$

95. What is the length of the conjugate axis of the hyperbola $\frac{x^2}{16} - \frac{y^2}{25} = 1$?

A. 4  
B. 5  
C. 8  
D. 10

96. Change $2y^2 - 4y - x - 2 = 0$ to standard form.

A. $x = (y - 1)^2 - 1$  
B. $x = (y - 1)^2 - 2$  
C. $x = 2(y - 1)^2 - 4$  
D. $x = 2(y - 1)^2 - 3$
97. A hyperbola with vertices (-2, 5) and (-2, -1), has an asymptote that passes through the point (2, 5). Determine an equation of the hyperbola.

A. \( \frac{(y-2)^2}{25} - \frac{(x+2)^2}{4} = 1 \)  
B. \( \frac{(y+2)^2}{9} - \frac{(x-2)^2}{16} = 1 \)
C. \( \frac{(y+2)^2}{25} - \frac{(x-2)^2}{4} = 1 \)  
D. \( \frac{(y-2)^2}{9} - \frac{(x+2)^2}{16} = 1 \)

98. The cross section of a drainage ditch is parabolic in shape, as shown in the diagram below. When the width of the water surface is 10 metres, the maximum depth of the water is 1.5 metres. Determine the width of the water, \( w \), when the maximum depth is 3 metres.

AUG 1999

99. Determine an equation for the set of all points \( P(x, y) \) such that the sum of the distances from \( P \) to \( A(2, 5) \) and from \( P \) to \( B(2, -7) \) is 20.

A. \( (x+2)^2 + (y+5)^2 + (x+2)^2 + (y-7)^2 = 400 \)
B. \( (x-2)^2 + (y-5)^2 + (x-2)^2 + (y+7)^2 = 400 \)
C. \( \sqrt{(x+2)^2 + (y+5)^2} + \sqrt{(x+2)^2 + (y-7)^2} = 20 \)
D. \( \sqrt{(x-2)^2 + (y-5)^2} + \sqrt{(x-2)^2 + (y+7)^2} = 20 \)

100. An ellipse, with centre in quadrant I, is tangent to both axes. The centre of the ellipse is on the line \( y = \frac{4}{3}x \). Which of the following could be an equation of this ellipse?

A. \( \frac{(x-3)^2}{16} + \frac{(y-4)^2}{9} = 1 \)  
B. \( \frac{(x-3)^2}{9} + \frac{(y-4)^2}{16} = 1 \)
C. \( \frac{(x-4)^2}{16} + \frac{(y-3)^2}{9} = 1 \)  
D. \( \frac{(x-4)^2}{9} + \frac{(y-3)^2}{16} = 1 \)
JAN 2000

101. Which conic is described by the equation $5x^2 - 3x + 2y - 7 = 0$?
   A. circle   B. ellipse   C. parabola   D. hyperbola

102. Change to standard form: $25x^2 + 4y^2 + 50x - 75 = 0$
   A. $\frac{(x + 1)^2}{2} + \frac{y^2}{12.5} = 1$   B. $\frac{(x + 1)^2}{3} + \frac{y^2}{37.5} = 1$   C. $\frac{(x + 1)^2}{4} + \frac{y^2}{25} = 1$   D. $\frac{(x + 1)^2}{5} + \frac{y^2}{62.5} = 1$

103. Determine an equation of the hyperbola which has asymptotes with slopes $\pm \frac{2}{3}$, and vertices at $\left(0, 2\right)$ and $\left(0, -2\right)$.
   A. $\frac{x^2}{9} - \frac{y^2}{4} = -1$   B. $\frac{x^2}{9} - \frac{y^2}{4} = 1$   C. $\frac{x^2}{4} - \frac{y^2}{9} = -1$   D. $\frac{x^2}{4} - \frac{y^2}{9} = 1$

104. The cross section of the roof of an indoor tennis court has a semi-elliptical shape. If the roof spans 86 m and has a maximum height of 30 m, find the height of the roof 20 m from the centre A.

   ![Diagram of a semi-elliptical roof]

   A. 16.05 m   B. 26.56 m   C. 27.32 m   D. 29.18 m

105. A child throws her beach ball with radius 28 cm into a basketball hoop with an inside diameter of 46 cm. The ball is too big and gets stuck, as shown in the diagram. What is the vertical distance $x$ from the top of the ball to the level of the hoop?
106. A point \( P(x, y) \) moves such that it is always the same distance from \( A(12, 0) \) as it is from \( B(3, 1) \). Determine an equation, in standard form, of this locus, and graph the conic on the grid below.

**APR 2000**

107. A hollow spherical ball has been sliced to form a bowl that measures 36 cm across the centre of the opening as shown in the diagram. If the bowl is 10 cm deep, determine the original radius of the spherical ball.
JUN 2000

108. Change $x^2 + 2x - 2y + 3 = 0$ to standard form.

A. $y = \frac{1}{2}(x + 1)^2 + 1$  
B. $y = \frac{1}{2}(x + 1)^2 + 2$  
C. $y = \frac{1}{2}(x + 1)^2 + \frac{1}{2}$  
D. $y = \frac{1}{2}(x + 1)^2 - 2$

109. A point $P(x, y)$ moves such that it is always the same distance from the point $A(5, 2)$ as it is from the line $x = -1$. Determine an equation of this

A. $\sqrt{(x - 5)^2 + (y - 2)^2} = \sqrt{(x + 1)^2 + (y - 0)^2}$  
B. $\sqrt{(x - 5)^2 + (y - 2)^2} = \sqrt{(x + 1)^2 + (y - y)^2}$  
C. $\sqrt{(x - 5)^2 + (y - 2)^2} = \sqrt{(x - x)^2 + (y + 1)^2}$  
D. $\sqrt{(x - 5)^2 + (y - 2)^2} = \sqrt{(x - 0)^2 + (y + 1)^2}$

110. A hyperbola with vertices $(-3, 8)$ and $(-3, -4)$ has asymptotes with slopes $\pm \frac{2}{3}$. Determine the equation of the hyperbola.

A. $\frac{(x - 3)^2}{9} - \frac{(y + 2)^2}{4} = -1$  
B. $\frac{(x + 3)^2}{9} - \frac{(y - 6)^2}{4} = -1$  
C. $\frac{(x + 3)^2}{16} - \frac{(y - 2)^2}{36} = -1$  
D. $\frac{(x + 3)^2}{81} - \frac{(y - 2)^2}{36} = -1$

111. A tunnel is semi-elliptical in shape. It has four traffic lanes, each 5 m wide, as well as two service walkways, each 2 m wide, as shown in the diagram. The tunnel has a height of 5 m at the edge of the roadway. Determine the height, $h$, of the tunnel at its highest point.

![Diagram of tunnel with dimensions and traffic lanes](image)

AUG 2000

112. Determine the coordinates of the point on the circle defined by $x^2 + y^2 = 5$ that is closest to the point $P(1,3)$.

A. $(1,2)$  
B. $(0.52, 2.17)$  
C. $(0.71, 2.12)$  
D. $(2.12, 0.71)$
113. A point \(P(x, y)\) moves such that it is always twice as far from the line \(y = -2\) as it is from the point \(R(0, 1)\). Determine an equation of this locus.

A. \(\sqrt{(x-0)^2 + (y-1)^2} = 2\sqrt{(x-x)^2 + (y+2)^2}\)
B. \(\sqrt{(x-0)^2 + (y-1)^2} = 2\sqrt{(x-0)^2 + (y+2)^2}\)
C. \(2\sqrt{(x-0)^2 + (y-1)^2} = \sqrt{(x-0)^2 + (y+2)^2}\)
D. \(2\sqrt{(x-0)^2 + (y-1)^2} = \sqrt{(x-0)^2 + (y+2)^2}\)

114. Determine the radius of the circle that passes through the points \((0, 5)\) and \((12, 0)\) and \((0, 0)\)

A. 5.5  
B. 6.5  
C. 7.5  
D. 8.5

115. A bridge over a river is supported by an arch in the shape of a rectangular hyperbola as shown in the diagram. The equation of the arch is \(y^2 - x^2 = 25\). If the maximum height of the arch above the water is 8 m, determine the width, \(w\), of the river.

![Diagram of a bridge arch](image)

JAN 2001

116. Which conic is represented by \(2x^2 + 3y^2 + 4x - 12y - 10 = 0\) ?

A. circle  
B. ellipse  
C. parabola  
D. hyperbola

117. Determine the vertex of the parabola \(x = 4(y - 3)^2 + 2\).

A. \((2, 3)\)  
B. \((2, -3)\)  
C. \((3, -2)\)  
D. \((3, 2)\)
118. The vertices of a hyperbola are \((-4, 3)\) and \((4, 3)\), and the slopes of its asymptotes are \(\pm \frac{1}{2}\). Determine an equation of the hyperbola.

A. \(\frac{x^2}{4} - \frac{(y-3)^2}{2} = 1\)  \hspace{1cm} B. \(\frac{x^2}{4} - \frac{(y-3)^2}{16} = 1\)  \hspace{1cm} C. \(\frac{x^2}{16} - \frac{(y-3)^2}{4} = 1\)  \hspace{1cm} D. \(\frac{x^2}{16} - \frac{(y-3)^2}{8} = 1\)

119. A suspension bridge has a cable in the shape of a parabola, as shown in the diagram below. The road passes through the vertex. The supporting towers are 60 metres high and 400 metres apart. A bird sits on the cable at a horizontal distance of 80 metres from a supporting tower. Determine the height of the bird above the road.

A. 9.6 m  \hspace{1cm} B. 21.6 m  \hspace{1cm} C. 24.0 m  \hspace{1cm} D. 36.0 m

120. Change \(4x^2 - 32x - 9y^2 - 36y - 116 = 0\) to standard form and graph on the grid below.
121. An arch is in the shape of a semi-circle. If the base of the arch measures 20 m, how high is the arch 3 m from the outside edge of the base?
   A. 7.14 m  
   B. 8.00 m  
   C. 9.27 m  
   D. 9.54 m

122. A circular pipe is filled with water to the level shown in the diagram. The water surface has a width of 30 cm and the space from the water surface to the top of the pipe is 5 cm. Determine the depth of the water.
   A. 25 cm  
   B. 40 cm  
   C. 45 cm  
   D. 50 cm

123. A cone has radius 8 cm and height 12 cm. Determine the radius, r, of a sphere that will just fit into the cone so that its centre is level with the top of the cone, as shown in the diagram.

124. Determine the length of the transverse axis of the hyperbola \( \frac{x^2}{9} - \frac{y^2}{16} = 1 \)
   A. 3  
   B. 4  
   C. 6  
   D. 8

125. Change \( 5x^2 + y^2 - 20x - 10 = 0 \) to standard form.
   A. \( \frac{(x-2)^2}{2} + \frac{y^2}{10} = 0 \)  
   B. \( \frac{(x-2)^2}{6} + \frac{y^2}{30} = 1 \)  
   C. \( \frac{(x-2)^2}{14} + \frac{y^2}{14} = 1 \)  
   D. \( \frac{(x-2)^2}{4} + \frac{y^2}{20} = 1 \)
126. A point \( P(x, y) \) moves such that the slope of the line through \( P \) and \( A(-2, 0) \) is always twice the slope of the line through \( P \) and \( B(2, 0) \). Determine an equation of this locus.

   A. \( \frac{y-0}{x+2} = 2 \left( \frac{y-0}{x-2} \right) \)
   B. \( \frac{y-0}{x-2} = 2 \left( \frac{y-0}{x+2} \right) \)
   C. \( \frac{y-0}{x+2} = 2 + \frac{y-0}{x-2} \)
   D. \( \frac{y-0}{x-2} = 2 + \frac{y-0}{x+2} \)

127. The vertical angle of a cone is 48°, as shown in the diagram. A sphere of radius 8 cm is dropped into the cone. Determine the shortest distance from the sphere to the vertex of the cone.

   A. 8.76 cm
   B. 9.97 cm
   C. 11.67 cm
   D. 11.96 cm

128. An ellipse which has vertices at \((-2, 2)\) and \((8, 2)\) is tangent to the \( x \)-axis

   a) Determine an equation of the ellipse.

   b) If \((6, y)\) is a point on the ellipse, determine all possible values for \(y\).

   c) Graph the conic on the grid below.