**RADICAL FUNCTIONS**

1. A function \( f(x) = \) is transformed to obtain \( g(x) = \sqrt{x-3} + 4 \). Which of the following transformations have occurred from \( f(x) \) to \( g(x) \)?

A. A vertical shift down 4 units and a horizontal shift left 3 units.
B. A vertical shift down 3 units and a horizontal shift left 4 units.
C. A vertical shift up 4 units and a horizontal shift right 3 units.
D. A vertical shift up 3 units and a horizontal shift right 4 units.

2. The equation that best describes the graph of the function shown is

A. \( f(x) = \sqrt{-x} - 2 \)
B. \( f(x) = -\sqrt{x} - 2 \)
C. \( f(x) = \sqrt{x} - 2 \)
D. \( f(x) = \sqrt{x} + 4 \)

3. Sketch the graph of \( y = -\sqrt{3(x - 1)} \).
4. Sketch the graph of \( y = -2\sqrt{x - 3} \). 

5. Sketch the graph of \( y = \sqrt{2x - 2} \).
6. Sketch the graph of the function \( y = \sqrt{-x} + 1 \).

Determine the value of \( x \) when \( y = 3 \).

7. The graph of the function \( y = f(x) \) is shown below. Sketch the graph of \( y = \sqrt{f(x)} \) and state the domain and range.
8. Given the graph of \( y = f(x) \) below, state the domain and range of \( y = \sqrt{f(x)} \).
10. Given the graph of $y = f(x)$ as shown, which of the following correctly shows the graph of $y = \sqrt{f(x)}$?
11. For each of the following functions, sketch the graphs of \( y = f(x) \) and \( y = \sqrt{f(x)} \).
   
   a) \( y = -x - 2 \)
   
   b) \( y = x^2 - 4 \)

   ![Graphs A and B]

12. Given \( f(x) = (x + 3)(x - 1) \), sketch the graphs of \( y = f(x) \) and \( y = \sqrt{f(x)} \).

   ![Graphs C and D]
13. The function \( f(x) = x^2 - x - 6 \). State the restrictions on the domain of \( y = \sqrt{f(x)} \).
   
   A. \( x = 3, -2 \)  
   B. \( -2 \leq x \leq 3 \)  
   C. \( x \leq -2 \) or \( x \geq 3 \)  
   D. \( x \geq 0 \)

14. Given the graph of \( y = f(x) \), sketch the graph of \( y = \sqrt{f(x)} \).

15. Given the graph of \( y = f(x) \), sketch the graph of \( y = \sqrt{f(x)} \).
16. Determine the range of the function \( y = \sqrt{3x - 9} + 2 \).
   
   A. \( y \geq 0 \)  
   B. \( y \geq 2 \)  
   C. \( y \geq 3 \)  
   D. \( y \geq 9 \)

17. State the coordinates of any invariant points when \( f(x) = \frac{1}{2}x - 3 \) is transformed to \( y = \sqrt{f(x)} \).

18. Determine the domain of the function \( y = \sqrt{-4x} \).
   
   A. \( x \geq 2 \)  
   B. \( x \leq 2 \)  
   C. \( x \geq 0 \)  
   D. \( x \leq 0 \)

19. Solve: \( 2\sqrt{2(x+4)} - 8 = 0 \)
   
   A. \(-4\)  
   B. \(4\)  
   C. \(12\)  
   D. \(128\)

20. Solve: \( \sqrt{x+4} = 3x \)
   
   A. \(-0.61, 0.72\)  
   B. \(-0.61\)  
   C. \(0.72\)  
   D. \(1.33\)

21. Determine the \( x \)-intercept of \( y = -2\sqrt{x+4} + 3 \), to the nearest hundredth.
RATIONAL FUNCTIONS

Sketch the graph of each of the functions in questions 22 to 33. Include the following key features of the function and indicate on your graph:

i) x and y intercepts;
ii) asymptote lines and their equations;
iii) point discontinuities (holes).

22. \( f(x) = \frac{3}{4x - 5} \)
23. \( f(x) = \frac{2x}{x + 2} \)

24. \( f(x) = \frac{x - 4}{x^2 - 3x - 4} \)
25. \( f(x) = \frac{x^2}{x^2 - x} \)

26. \( f(x) = \frac{1}{x^2 + 2x - 8} \)
27. \( f(x) = \frac{x^2 - 4}{x^2 - 2x} \)

28. \( f(x) = \frac{3x}{x^2 + 2x - 8} \)
29. \( f(x) = \frac{x + 3}{x^2 - 9} \)

30. \( f(x) = \frac{x^3 - 2x^2}{x^2 - 4} \)
31. \( f(x) = \frac{3}{x^2 + 1} \)

32. \( f(x) = \frac{x - 3}{x - 2} \)
33. \( f(x) = \frac{x - 2}{(2x - 3)(x - 2)} \)

34. The function \( f(x) \) is transformed. A new function, \( y = \frac{1}{f(x)} \) is created that does not have any vertical asymptotes. What can you conclude about the original function \( f(x) \)?

35. The graph of the function below can be expressed in the form \( \frac{ax}{x^2 + bx + c} \). Determine the values of \( a, b, \) and \( c \).
36. Determine the coordinates of the point of discontinuity on the graph of \( f(x) = \frac{2x^2 - 15x + 7}{x - 7} \).

37. For the graph of \( y = \frac{3x + 7}{2x + 5} \), determine the equation of the horizontal asymptote and the range.

38. Consider the graphs of \( f(x) = \frac{x^2 - x - 6}{x^2 - 9} \) and \( g(x) = \frac{x}{x^2 - 9} \). Use your knowledge of rational functions to outline the similarities and differences between these two graphs.

39. Determine the equations of all asymptotes for the graph of \( y + 2 = \frac{1}{x - 1} \).

   A. \( x = -1, \ y = 2 \)  B. \( x = 1, \ y = -2 \)  C. \( x = 1, \ y = 2 \)  D. \( x = -1, \ y = -2 \)
40. Which of the following best represents the graph of the rational function \( y = \frac{x}{x^2 - 4x} \)?

- A.
- B.
- C.
- D.

41. The function \( f(x) = \frac{x^2 - 9}{x^2 - 5x + 6} \) has a vertical asymptote at \( x = a \) and a hole at \( x = b \). Determine possible values for \( a \) and \( b \).
42. Which of the following equations best matches the graph shown?

A. \( f(x) = \frac{2}{(x-2)(x+1)} \)  
B. \( f(x) = \frac{2}{(x+2)(x-1)} \)  
C. \((x-2)(x+1)\)  
D. \((x+2)(x-1)\)

43. Determine the \(x\)-intercepts of the function \( \frac{x^2 + 5x + 4}{x^2 - 16} \).

44. Which of the following functions has a point discontinuity at \( x = 3 \)?

A. \( f(x) = \frac{x-3}{x^2 - 6x - 12} \)  
B. \( f(x) = \frac{x-3}{2x^2 - 2x - 12} \)  
C. \( f(x) = \frac{x+3}{x^2 - 6x - 12} \)  
D. \( f(x) = \frac{x+3}{x^2 - 6x + 9} \)
45. Determine a possible equation for the graph of a rational function with the following characteristics:

- a vertical asymptote with equation $x = -1$
- a horizontal asymptote with equation $y = 1$
- a point discontinuity at $(-2, 3)$

A. $f(x) = \frac{(x + 1)(x + 2)}{(x - 1)(x + 2)}$ 
B. $f(x) = \frac{(x - 1)(x + 2)}{(x + 1)(x + 2)}$
C. $f(x) = \frac{(x - 1)(x - 2)}{(x + 1)(x - 2)}$ 
D. $f(x) = \frac{(x + 1)(x - 2)}{(x - 1)(x - 2)}$

46. Determine a possible equation for the graph of a rational function with the following characteristics:

- a vertical asymptote with equation $x = 2$
- a horizontal asymptote with equation $y = 8$
- a point discontinuity at $(3, 9)$

A. $f(x) = \frac{x - 3}{(x - 3)(x - 2)} + 8$ 
B. $f(x) = \frac{x - 2}{(x - 3)(x + 2)} + 8$
C. $f(x) = \frac{x + 3}{(x + 2)(x + 3)} + 8$ 
D. $f(x) = -\frac{x - 3}{(x + 3)(x - 2)} - 8$

47. Which of the following is true about the two functions below?

\[ f(x) = \frac{(x + 2)(x - 2)}{x - 2} \quad g(x) = \frac{(x - 2)(x + 1)}{(x + 2)(x - 2)} \]

A. Both have a point discontinuity (hole) when $x = 2$.
B. Both have the same vertical asymptote.
C. Both have the same horizontal asymptote.
D. Both have the same $y$-intercept.
48. Match each graph below with its possible equation:

A. \( f(x) = \frac{9}{x^2 - 4} \)  
B. \( f(x) = \frac{1}{x + 4} \)  
C. \( f(x) = -\frac{1}{x + 4} \)  
D. \( f(x) = \frac{1}{x^2 + 4} \)  
E. \( f(x) = \frac{-3x - 9}{x + 4} \)  
F. \( f(x) = \frac{9}{x^2 + 6x + 8} \)  
G. \( f(x) = \frac{1}{(x - 4)^2} \)  
H. \( f(x) = \frac{x}{-3(x + 4)} \)
49. Match each graph below with its correct description.
   A. The graph of this function has a vertical asymptote at \( x = -1 \).
   B. The graph of this function has a point of discontinuity (hole) at \( x = 3 \).
   C. The graph of this function has a horizontal asymptote at \( y = 4 \).
   D. The domain of this function is \( x \in \mathbb{R} \).

   \[
   f(x) = \frac{4}{x^2 + 1} \quad f(x) = \frac{4x}{x + 3} \quad f(x) = \frac{4(x - 3)(x + 2)}{x - 3} \quad f(x) = \frac{4(x - 3)}{(x + 3)(x + 1)}
   \]

50. Determine the coordinates of the point discontinuity (hole) for the graph of the function

   \[
   y = \frac{(2 - x)(x - 3)}{x - 2}.
   \]

51. The function \( f(x) = x^2 - x - 6 \). State the domain and range of \( y = \frac{1}{f(x)} \).