REVIEW OF SUM AND DIFFERENCE AND DOUBLE ANGLE IDENTITIES ANSWER KEY

1a. $\sin^2(2.5)+\cos^2(2.5) = 1$ (Pythagorean Identity, θ = 2.5 radians)

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b. $\cos(-\theta)\sec(-\theta)-\csc(\theta)\sin(-\theta)$

$\cos(-\theta)=\cos\theta$, by even symmetry $\therefore \sec(-\theta)=\sec(\theta)$

and $\sin(-\theta)=-\sin(\theta)$, by odd symmetry

$\therefore \cos(-\theta)\sec(-\theta)-\csc(\theta)\sin(-\theta)=\cos(\theta)\sec(\theta)-\csc(\theta)(-\sin(\theta))=1-(-1)=2$

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c. $\sin 160^\circ\cos 20^\circ+\cos 160^\circ\sin 20^\circ=\sin(160^\circ+20^\circ)=\sin(180^\circ)=0$

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d. $\dfrac{\sin 4\theta}{2\sin 2\theta}=\dfrac{2\sin 2\theta\cos 2\theta}{2\sin 2\theta}=\cos 2\theta$

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e. $\sin\theta\csc\theta+\dfrac{\sin\theta}{\cos\theta\cot\theta}=1+\dfrac{\sin\theta}{\left(\dfrac{\cos\theta}{1}\right)\left(\dfrac{\cos\theta}{\sin\theta}\right)}=1+\dfrac{\sin\theta}{\left(\dfrac{\cos^2\theta}{\sin\theta}\right)}=1+\dfrac{\sin^2\theta}{\cos^2\theta}=1+\tan^2\theta=\sec^2\theta$

2a. $1-2\sin^2(1.5)=\cos(3)$ (That is the cosine of 3 radians)

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b. $\sin(0.8)\cos(0.8)=\dfrac{\sin(1.6)}{2}$

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c. $2\sin^2(0.75)-1=-\cos(1.5)$

3a. $\sin(2\beta)=2\sin\beta\cos\beta=2\left(-\dfrac{12}{13}\right)\left(-\dfrac{5}{13}\right)=\dfrac{120}{169}$

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b. $\cos(\theta+\pi)=\cos\theta\cos\pi-\sin\theta\sin\pi=(\cos\theta)(-1)-(\sin\theta)(0)=-\cos\theta$

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c. $\sin(\beta-\theta)=\sin\beta\cos\theta-\cos\beta\sin\theta=\left(-\dfrac{12}{13}\right)\left(-\dfrac{3}{5}\right)-\left(-\dfrac{5}{13}\right)\left(\dfrac{4}{5}\right)=\dfrac{56}{65}$

4c. $\sin 3\theta$ $\qquad\qquad$ $3\sin\theta\cos^2\theta-\sin^3\theta$

$=\sin(\theta+2\theta)$

$=\sin\theta\cos 2\theta+\cos\theta\sin 2\theta$

$=\sin\theta\left(\cos^2\theta-\sin^2\theta\right)+\cos\theta(2\sin\theta\cos\theta)$

$=\sin\theta\cos^2\theta-\sin^3\theta+2\sin\theta\cos^2\theta$

$=3\sin\theta\cos^2\theta-\sin^3\theta$
