AP CALCULUS PROBLEM SET #11 INTEGRATION III FTC ANSWER KEY

1. a) Inflection points at $x = -2$ and $x = 0$

   b) $f(-4) = 5 + \int_0^{-4} g(x)dx = 2\pi - 3$

   $f(4) = 5 + \int_0^4 \left(5e^{-x/3} - 3\right)dx = 8 - 15e^{-4/3}$

   c) Abs. maximum at $x = 3\ln\left(\frac{5}{3}\right)$

2. a) $g(3) = 5 + \int_0^3 g'(x)dx = \frac{13}{2} + \pi$

   $f(-2) = 5 + \int_0^{-2} g'(x)dx = 5 - \pi$

   b) Inflection points at $x = 0$, 2 and 3

   c) $h'(x) = g'(x) - x = 0$ when $x = \sqrt{2}$ and $x = 3$

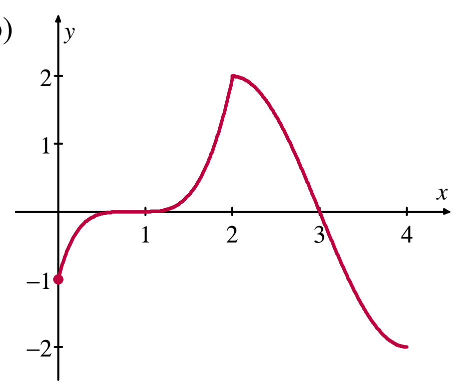
   h has a relative max at $x = \sqrt{2}$,

   and neither a maximum nor minimum

   at $x = 3$

3. a) Relative maximum at $x = 2$

   b)



   c) $g'(x) = f(x) = 0$ at $x = 1, 3$

   g has rel. min. at $x = 1$,

   rel. max. at $x = 3$

   d) g has an inflection point at $x = 2$.

4. a) $g(0) = 4.5$, $g'(0) = 1$

   b) g has rel. max. at $x = 3$

   c) g has abs. min. at $x = -4$, $g = -1$

   d) $x = -3, 1, 2$

5. a) f is increasing on $[-3,-2]$

   b) $x = 0$ and $x = 2$

   c) $y = -2x + 3$

   d) $f(-3) = 9/2$, $f(4) = -5 + 2\pi$

6. a) $6\pi^2 - \left[2\sin\left(\frac{x}{2}\right)\right]_{x=-2\pi}^{x=4\pi} = 6\pi^2$

   b) $f'(x) = g'(x) + \frac{1}{2}\sin\left(\frac{x}{2}\right)$

   $= \begin{cases} 1 + \frac{1}{2}\sin\left(\frac{x}{2}\right) & \text{for } -2\pi < x < 0 \\ -\frac{1}{2} + \frac{1}{2}\sin\left(\frac{x}{2}\right) & \text{for } 0 < x < 4\pi \end{cases}$

   $f'(x)$ does not exist at $x = 0$.

   For $-2\pi < x < 0$, $f'(x) \neq 0$.

   For $0 < x < 4\pi$, $f'(x) = 0$ when $x = \pi$.

   f has critical points at $x = 0$ and $x = \pi$.

   c) $h'(x) = g(3x) \cdot 3$

   $h'\left(-\frac{\pi}{3}\right) = 3g(-\pi) = 3\pi$

7. a) $g(4) = 3$, $g'(4) = 0$, $g''(4) = -2$

   b) g has rel. min. at $x = -1$

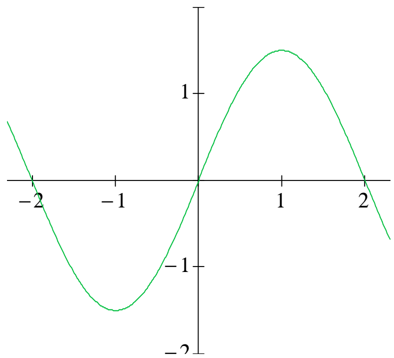
   c) at $x = 108$, $y - 44 = 2(x - 108)$

8. a) $g(-1) = -1.5$, $g'(-1) = 0$, $g''(-1) = 3$

b) g is increasing on $-1 < x < 1$ since $g'(x) = f(x) > 0$

c) g is concave down on $0 < x < 2$ since $g''(x) = f'(x) < 0$

d)



9. a) *i*) $5x^3 + 40 = \int_c^x f(t)dt$

$$15x^2 = f(x) \;\therefore\; f(t) = 15t^2$$

ii) $5x^3 + 40 = \int_c^x 15t^2 dt$

$$= 5t^3\Big]_c^x = 5x^3 - 5c^3$$

$$-5c^3 = 40, \;\therefore c = -2$$

b) $F(x) = \int_x^3 \sqrt{1+t^{16}}\,dt = -\int_3^x \sqrt{1+t^{16}}\,dt$

$$F'(x) = -\sqrt{1+x^{16}}$$

10. a) $0 \le \left(\frac{x}{2} + 3\right) \le 5$

$$-6 \le x \le 4$$

b) $h'(x) = f\left(\frac{x}{2} + 3\right)\left(\frac{1}{2}\right)$

$$h'(2) = f(4)\left(\frac{1}{2}\right) = -\frac{3}{2}$$

c) $h(-6) = \int_0^0 f(t)dt = 0$

$$h(4) = \int_0^5 f(t)dt < 0$$

Since area below x axis is greater than the area above, min @ $x = 4$.

11 a) f has a relative minimum at 1.

b) By the Mean Value Theorem, there is at least one value c, $-1 < c < 1$, such that $f''(c) = 0$.

c) $h'(x) = \frac{1}{f(x)} f'(x)$

$$h'(3) = \frac{1}{f(3)}\left(f'(3)\right) = \left(\frac{1}{7}\right)\left(\frac{1}{2}\right) = \frac{1}{14}$$

d) $\int_{-2}^{3} f'(g(x))g'(x)dx = \left[f(g(x))\right]_{-2}^{3}$

$$= f(g(3)) - f(g(-2))$$

$$= f(1) - f(-1) = 2 - 8 = -6$$