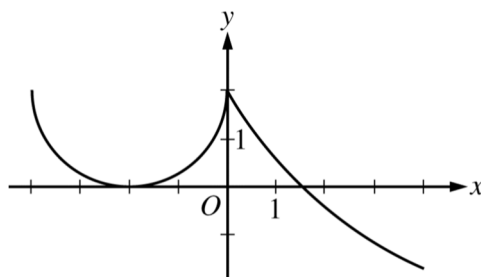


AP CALCULUS PROBLEM SET #11 INTEGRATION III FTC

(2009-6)

1.



Graph of f'

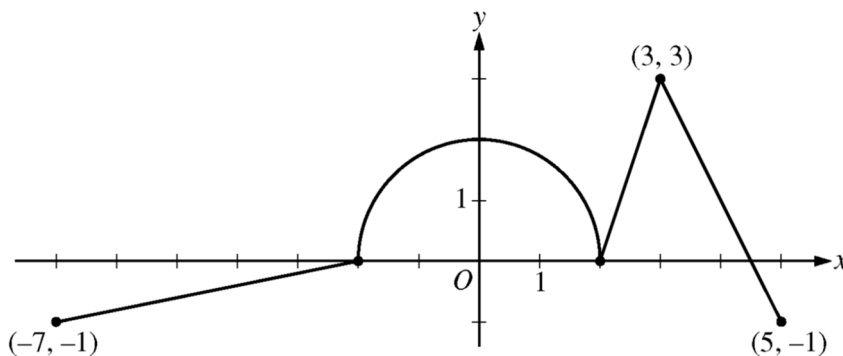
The derivative of a function f is defined by $f'(x) = \begin{cases} g(x) & \text{for } -4 \leq x \leq 0 \\ 5e^{-x/3} - 3 & \text{for } 0 < x \leq 4 \end{cases}$.

The graph of the continuous function $f'(x)$, shown in the figure above, has x -intercepts at $x = -2$ and $x = 3 \ln\left(\frac{5}{3}\right)$. The graph of g on $-4 \leq x \leq 0$ consists of a semi-circle, and $f(0) = 5$.

- For $-4 < x < 4$, find all values of x at which the graph of f has a point of inflection. Justify your answer.
- Find $f(-4)$ and $f(4)$.
- For $-4 \leq x \leq 4$, find the value of x at which f has an absolute maximum. Justify your answer.

(2010-5)

2.



Graph of g'

The function g is defined and differentiable on the closed interval $[-7, 5]$ and satisfies $g(0) = 5$.

The graph of $y = g'(x)$, the derivative of g , consists of a semi-circle and three line segments as shown on the figure above.

- Find $g(3)$ and $g(-2)$.
- Find the x -coordinate of each point of inflection of the graph of $y = g(x)$ on the interval $-7 < x < 5$. Explain your reasoning.
- The function h is defined by $h(x) = g(x) - \frac{1}{2}x^2$. Find the x -coordinate of each critical point of h , where $-7 < x < 5$, and classify each critical point as the location of a relative minimum, relative maximum, or neither a minimum nor a maximum. Explain your reasoning.

(2005-4)

3.

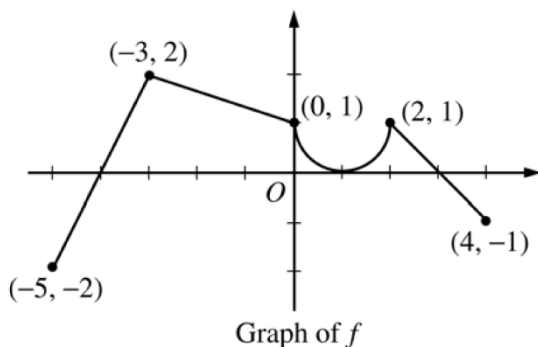
x	0	$0 < x < 1$	1	$1 < x < 2$	2	$2 < x < 3$	3	$3 < x < 4$
$f(x)$	-1	Negative	0	Positive	2	Positive	0	Negative
$f'(x)$	4	Positive	0	Positive	DNE	Negative	-3	Negative
$f''(x)$	-2	Negative	0	Positive	DNE	Negative	0	Positive

Let f be a function that is continuous on the interval $[0, 4)$. The function f is twice differential except at $x = 2$. The function f and its derivatives have the properties indicated in the table above, where DNE indicates that the derivatives of f do not exist at $x = 2$.

- (a) For $0 < x < 4$, find all values of x at which f has a relative extremum. Determine whether f has a relative maximum or a relative minimum at each of these values. Justify your answer.
- (b) On the axes provided, sketch the graph of a function that has all the characteristics of f .
- (c) Let g be the function defined by $g(x) = \int_1^x f(t)dt$ on the open interval $(0, 4)$. For $0 < x < 4$, find all values of x at which g has a relative extremum. Determine whether g has relative maximum or a relative minimum at each of these values. Justify your answer.
- (d) For the function defined in part (c), find all values of x , for $0 < x < 4$, at which the graph of g has a point of inflection. Justify your answer.

(2004-5)

4.

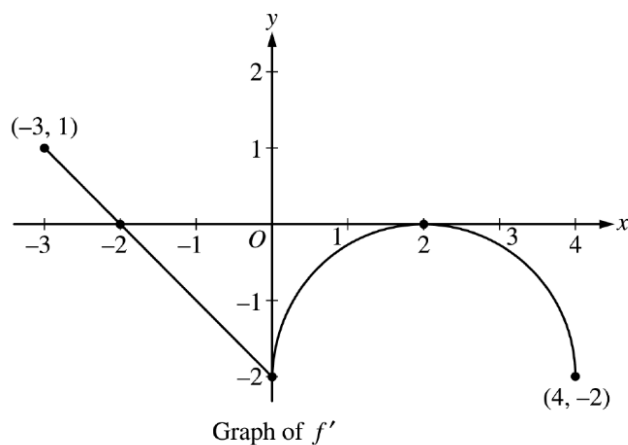


The graph of the function f shown above consists of a semicircle and three line segments. Let g be the function given by $g(x) = \int_{-3}^x f(t)dt$

- (a) Find $g(0)$ and $g'(0)$.
- (b) Find all values of x in the open interval $(-5, 4)$ at which g attains a relative maximum. Justify your answer.
- (c) Find the absolute minimum value of g on the closed interval $[-5, 4]$. Justify your answer.
- (d) Find all values of x in the open interval $(-5, 4)$. At which the graph of g has a point of inflection.

(2003-4)

5.

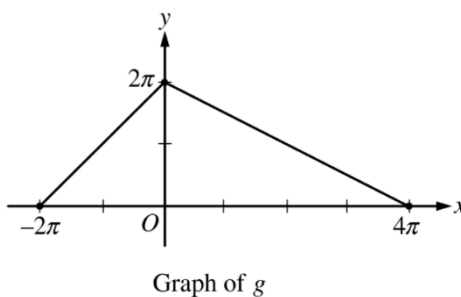


Let f be a function defined on the closed interval $-3 \leq x \leq 4$ with $f(0) = 3$. The graph of f' , the derivative of f , consists of one line segment and a semicircle as shown.

- (a) On what intervals, if any, is f increasing? Justify your answer.
- (b) Find the x -coordinate of each point of inflection of the graph of f on the open interval $-3 < x < 4$. Justify your answer.
- (c) Find an equation for the line tangent to the graph of f at the point $(0, 3)$
- (d) Find $f(-3)$ and $f(4)$. Show the work that leads to your answers

(2011(B)-6)

6.



Let g be the piecewise linear function defined on $[-2\pi, 4\pi]$ whose graph is given above, and let

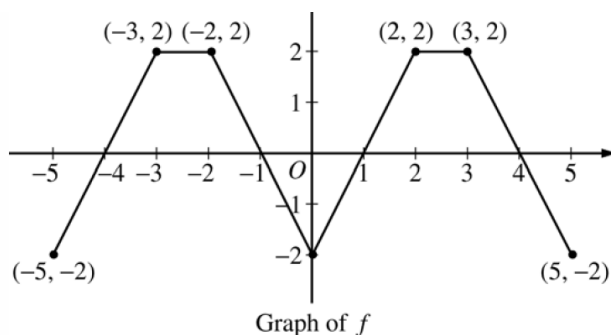
$$f(x) = g(x) - \cos\left(\frac{x}{2}\right)$$

- (a) Find $\int_{-2\pi}^{4\pi} f(x) dx$. Show the computations that lead to your answer.
- (b) Find all x -values in the open interval $(-2\pi, 4\pi)$ for which f has a critical point.
- (c) Let $h(x) = \int_0^{3x} g(t) dt$. Find $h'\left(-\frac{\pi}{3}\right)$.

OPTIONAL

(2006-3)

7.



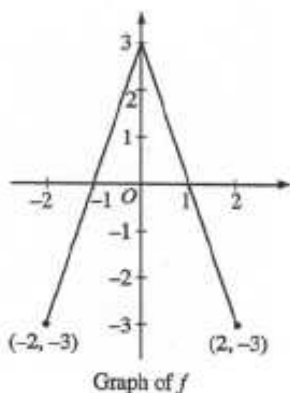
The graph of the function f shown above consists of six line segments.

Let g be the function given by $g(x) = \int_0^x f(t) dt$.

- Find $g(4)$, $g'(4)$, $g''(4)$.
- Does g have a relative minimum, a relative maximum, or neither at $x = 1$? Justify your answer.
- Suppose f is defined for all real numbers and is periodic with a period of length 5. The graph above shows two periods of f . Given that $g(5) = 2$, find $g(10)$ and write an equation of the line tangent to the graph of g at $x = 108$.

(2002-4)

8.



The graph of the function f shown above consists of two line segments.

Let g be the function given by $g(x) = \int_0^x f(t) dt$

- Find $g(1)$, $g'(1)$, $g''(1)$.
- For what values of x in the open interval $(-2, 2)$ is g increasing? Explain your reasoning.
- For what values of x in the open interval $(-2, 2)$ is the graph of g concave down? Explain your reasoning.
- On the axes provided, sketch the graph of g on the closed interval $[-2, 2]$

(76-6)

9.

(a) Given $5x^3 + 40 = \int_c^x f(t) dt$.

(i) Find $f(x)$

(ii) Find the value of c .

(b) If $F(x) = \int_x^3 \sqrt{1+t^{16}} dt$, find $F'(x)$.

(95BC-6)

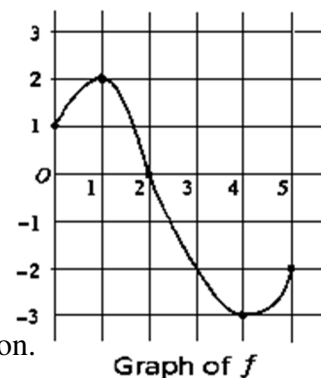
10. Let f be a function whose domain is the closed interval $[0, 5]$.

The graph of f is shown. Let $h(x) = \int_0^{\frac{x}{2}+3} f(t) dt$.

(a) Find the domain of h .

(b) Find $h'(2)$.

(c) At what x is $h(x)$ a minimum? Show the analysis that leads to your conclusion.



(2014-5)

11.

x	-2	$-2 < x < -1$	-1	$-1 < x < 1$	1	$1 < x < 3$	3
$f(x)$	12	Positive	8	Positive	2	Positive	7
$f'(x)$	-5	Negative	0	Negative	0	Positive	$\frac{1}{2}$
$g(x)$	-1	Negative	0	Positive	3	Positive	1
$g'(x)$	2	Positive	$\frac{3}{2}$	Positive	0	Negative	-2

The twice-differentiable functions f and g are defined for all real numbers x . Values of f , f' , g , and g' for various values of x are given in the table above.

(a) Find the x -coordinate of each relative minimum of f on the interval $[-2, 3]$. Justify your answers.

(b) Explain why there must be a value c , for $-1 < c < 1$, such that $f''(c) = 0$.

(c) The function h is defined by $h(x) = \ln f(x)$. Find $h'(3)$.

Show the computations that lead to your answer.

(d) Evaluate $\int_{-2}^3 f'(g(x))g'(x)dx$