

AP CALCULUS PROBLEM SETS #12 VOLUMES ANSWER KEY

1. a)  $x = -1, k = -e$

b)  $V = \int_{-1}^0 (e^{-x} - (-ex)) dx = \frac{e}{2} - 1$

c)  $\pi \int_{-1}^0 [(e^{-x})^2 - (-ex)^2] dx$

2. a)  $R = \int_0^4 \sqrt{x} dx = \frac{16}{3}$  units<sup>2</sup>

b)  $h = 4^{\frac{2}{3}} \approx 2.520$

c)  $V = \pi \int_0^4 (\sqrt{x})^2 dx = 8\pi$  units<sup>3</sup>

d)  $k = \sqrt{8} \approx 2.828$

3. a)  $R = \int_0^{0.902155} \tan x dx + \int_{0.902155}^{\sqrt[3]{2}} (2 - x^3) dx$   
 $= 0.729$  units<sup>2</sup>

b)  $S = \int_0^{0.902155} (2 - x^3 - \tan x) dx = 1.160$  units<sup>2</sup>

c)  $V = \pi \int_0^{0.902155} [(2 - x^3)^2 - \tan^2 x] dx$   
 $= 8.331$  units<sup>3</sup>

4. a)  $\int_{0.5}^1 (e^x - \ln x) dx = 1.223$

b)  $\pi \int_{0.5}^1 [(4 - \ln x)^2 - (4 - e^x)^2] dx = 23.609$

c) Abs. max =  $h(1) = 2.718$ , Abs. min = 2.330

5. a)  $A = \int_0^9 [6 - 2\sqrt{x}] dx = 18$

b)  $V = \pi \int_0^9 [(7 - 2\sqrt{x})^2 - (7 - 6)^2] dx$

c)  $V = \int_0^6 \frac{3}{16} y^4 dy$

6. a)  $A = \int_0^2 [\sin(\pi x) - (x^3 - 4x)] dx = 4$

b)  $A = \int_{0.5391889}^{1.6751309} [-2 - (x^3 - 4x)] dx$

c)  $V = \int_0^2 [\sin(\pi x) - (x^3 - 4x)]^2 dx = 9.978$

d)  $V = \int_0^2 (3 - x) [\sin(\pi x) - (x^3 - 4x)] dx$   
 $= 8.369$

7. a)  $150\pi$  ft<sup>3</sup> or 471.238 ft<sup>3</sup>

b) 59 minutes

c)  $\frac{dh}{dt} = \frac{12}{25\pi}$  ft/min or 0.152 ft/min

8. a)  $V = 2\pi \int_0^\infty x e^{-x^2} dx$

$$= 2\pi \lim_{b \rightarrow \infty} \int_0^b x e^{-x^2} dx$$

$$= 2\pi \lim_{b \rightarrow \infty} \left[ -\frac{1}{2} e^{-x^2} \right]_0^b$$

$$= 2\pi \lim_{b \rightarrow \infty} \left( -\frac{1}{2} e^{-b^2} + \frac{1}{2} e^0 \right)$$

$$= 2\pi \left( \frac{1}{2} \right) = \pi$$

b) Inflection point for  $h(x)$  and maximum of

$$A(w) \text{ occur at } x = \frac{1}{\sqrt{2}}$$

$$9. \text{ a) } V = \pi \int_3^5 (x^2 - 9) dx = \frac{44\pi}{3}$$

$$\text{b) } V = 2\pi \int_3^5 (x+1) \left( \sqrt{x^2 - 9} \right) dx \text{ (Shells) or}$$

$$\pi \int_0^4 \left[ 36 - \left( \sqrt{9 + y^2} + 1 \right)^2 \right] dy \text{ (Washers)}$$

$$10. \text{ a) } A = \int_{0.15859}^{3.14619} [\ln(x) - (x-2)] dx = 1.949$$

$$\text{b) } V = \pi \int_{0.15859}^{3.14619} \left[ (\ln(x) + 3)^2 - (x - 2 + 3)^2 \right] dx \\ = 34.198$$

$$\text{c) } V = \int_{0.15859-2}^{3.14619-2} \left[ (y+2)^2 - (e^y)^2 \right] dy$$