

## AP CALCULUS PROBLEM SET #12 VOLUMES

(92-5)

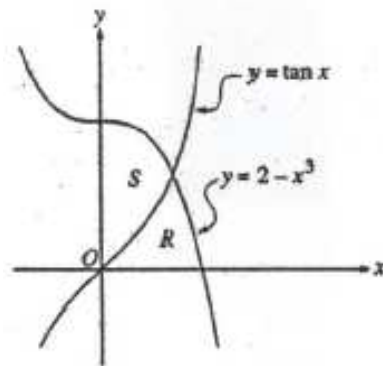
1. Let  $f$  be the function given by  $f(x) = e^{-x}$  and let  $g$  be the function given by  $g(x) = kx$ , where  $k$  is the nonzero constant such that the graph of  $f$  is tangent to the graph of  $g$ .
  - (a) Find the  $x$ -coordinate of the point of tangency and the value of  $k$ .
  - (b) Let  $R$  be the region enclosed by the  $y$ -axis and the graphs of  $f$  and  $g$ . Using the results found in part (a), determine the area of  $R$ .
  - (c) Set up, but do not integrate, an integral expression in terms of a single variable for the volume of the solid generated by revolving the region  $R$ , given in part (b), about the  $x$ -axis.

(98-1)

2. Let  $R$  be the region bounded by the  $x$ -axis, the graph of  $y = \sqrt{x}$ , and the line  $x = 4$ .
  - (a) Find the area of the region  $R$ .
  - (b) Find the value of  $h$  such that the vertical line  $x = h$  divides the region  $R$  into two regions of equal area.
  - (c) Find the volume of the solid generated when  $R$  is revolved about the  $x$ -axis.
  - (d) The vertical line  $x = k$  divides the region  $R$  into two regions such that when these two regions are revolved about the  $x$ -axis, they generate solids with equal volumes. Find the value of  $k$ .

(2001-1)

3. Let  $R$  and  $S$  be the regions in the first quadrant shown in the figure shown. The region  $R$  is bounded by the  $x$ -axis and the graphs of  $y = 2 - x^3$  and  $y = \tan x$ . The region  $S$  is bounded by the  $y$ -axis and the graphs of  $y = 2 - x^3$  and  $y = \tan x$ .



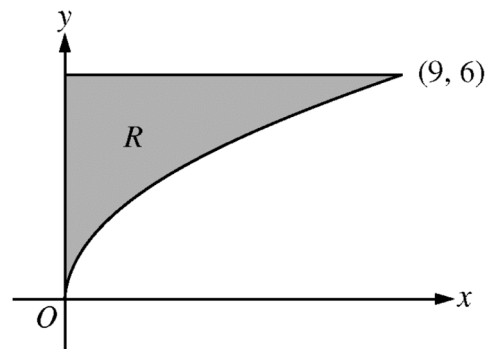
- (a) Find the area of  $R$ .
- (b) Find the area of  $S$ .
- (c) Find the volume of the solid generated when  $S$  is revolved about the  $x$ -axis.

(2002- 1)

4. Let  $f$  and  $g$  be the functions given by  $f(x) = e^x$  and  $g(x) = \ln x$ .
  - (a) Find the area of the region enclosed by the graphs of  $f$  and  $g$  between  $x = \frac{1}{2}$  and  $x = 1$ .
  - (b) Find the volume of the solid generated when the region enclosed by the graphs of  $f$  and  $g$  between  $x = \frac{1}{2}$  and  $x = 1$  is revolved about the line  $y = 4$ .
  - (c) Let  $h$  be the function given by  $h(x) = f(x) - g(x)$ . Find the absolute minimum value of  $h(x)$  on the closed interval  $\frac{1}{2} \leq x \leq 1$ , and find the absolute maximum value of  $h(x)$  on the closed interval  $\frac{1}{2} \leq x \leq 1$ . Show the analysis that leads to your answers.

(2010-4)

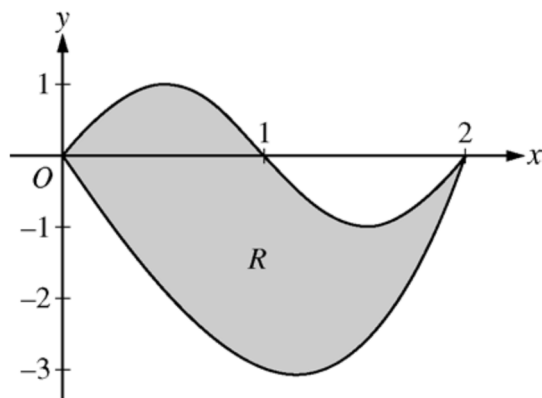
5. Let  $R$  be the region in the first quadrant bounded by the graphs of  $y = 2\sqrt{x}$ , the horizontal line  $y = 6$ , and the  $y$ -axis, as shown.



- (a) Find the area of  $R$ .
- (b) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when  $R$  is rotated about the horizontal line  $y = 7$ .
- (c) Region  $R$  is the base of a solid. For each  $y$ , where  $0 \leq y \leq 6$ , the cross section of the solid taken perpendicular to the  $y$ -axis is a rectangle whose height is 3 times the length of its base in region  $R$ . Write, but do not evaluate, an integral expression that gives the volume of the solid.

(2008-1)

6. Let  $R$  be the region in the first quadrant bounded by the graphs of  $y = \sin(\pi x)$  and  $y = x^3 - 4x$ , as shown in the figure.

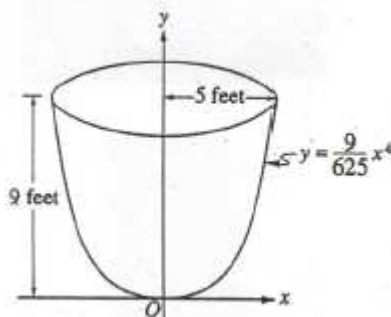


- (a) Find the area of  $R$ .
- (b) The horizontal line  $y = -2$  splits the region  $R$  into two parts. Write, but do not evaluate, an integral expression for the area of the part of  $R$  that is below this horizontal line.
- (c) Region  $R$  is the base of a solid. For this solid, each cross section of the solid taken perpendicular to the  $x$ -axis is a square. Find the volume of this solid.
- (d) The region  $R$  models the surface of a small pond. At all points in  $R$  at a distance  $x$  from the  $y$ -axis, the depth of the water is given by  $h(x) = 3 - x$ . Find the volume of water in the pond.

### OPTIONAL

(96-5)

7.

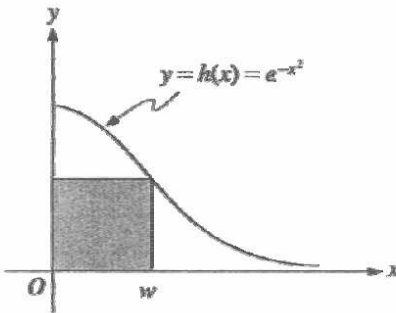


An oil storage tank has the shape shown above, obtained by revolving the curve  $y = \frac{9}{625}x^4$  from  $x = 0$  to  $x = 5$  about the  $y$ -axis, where  $x$  and  $y$  are measured in feet. Oil flows into the tank at the constant rate of 8 cubic feet per minute.

- (a) Find the volume of the tank. Indicate units of measure.
- (b) To the nearest minute, how long would it take to fill the tank if the tank was empty initially?
- (c) Let  $h$  be the depth, in feet, of oil in the tank. How fast is the depth of the oil in the tank increasing when  $h = 4$ ? Indicate units of measure.

(96BC-1)

8. Consider the graph of the function  $h$  given by  $h(x) = e^{-x^2}$  for  $0 \leq x < \infty$



- (a) Let  $R$  be the unbounded region in the first quadrant below the graph of  $h$ . Find the volume of the solid generated when  $R$  is revolved about the  $y$ -axis.
- (b) Let  $A(w)$  be the area of the shaded rectangle shown in the figure above. Show that  $A(w)$  has its maximum when  $w$  is the  $x$ -coordinate of the point of inflection of the graph of  $h$ .

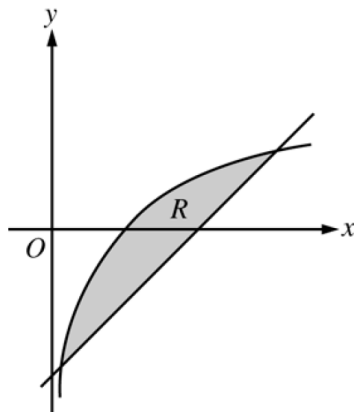
(88-3)

9. Let  $R$  be the region in the first quadrant enclosed by the hyperbola  $x^2 - y^2 = 9$ , the  $x$ -axis, and the line  $x = 5$ .

- (a) Find the volume of the solid generated by revolving  $R$  about the  $x$ -axis.
- (b) Set up, but do not integrate, an integral expression in terms of a single variable for the volume of the solid generated when  $R$  is revolved about the line  $x = -1$ .

(2006-1)

10.



Let  $R$  be the shaded region bounded by the graphs of  $y = \ln x$  and  $y = x - 2$ , as shown above.

- (a) Find the area of  $R$ .
- (b) Find the volume of the solid generated when  $R$  is rotated about the horizontal line  $x = -3$ .
- (c) Write, but do not evaluate, an integral expression that can be used to find the volume of the solid generated when  $R$  is rotated about the  $y$ -axis.