

AP PROBLEM SET #1

DERIVATIVES

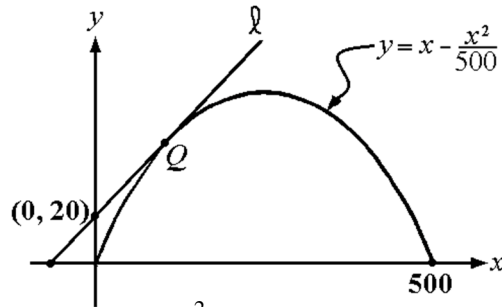
(79-1)

1. Given the function f defined by $f(x) = 2x^3 - 3x^2 - 12x + 20$.

- (a) Find the zeros of f .
- (b) Write an equation of the line normal to the graph of f at $x = 0$.
- (c) Find the x and y coordinates of all points on the graph of f where the line tangent to the graph is parallel to the x -axis.

(96-6)

2.



Line ℓ is tangent to the graph of $y = x - \frac{x^2}{500}$ at the point Q , as shown in the figure above.

- (a) Find the x -coordinate of point Q .
- (b) Write an equation for line ℓ .
- (c) Suppose the graph of $y = x - \frac{x^2}{500}$ shown in the figure, where x and y are measured in feet, represents a hill. There is a 50-foot tree growing vertically at the top of the hill. Does a spotlight at point P directed along line ℓ shine on any part of the tree? Show the work that leads to your conclusion.

(91-4)

3. Let f be the function given by $f(x) = \frac{|x| - 2}{x - 2}$.

- (a) Find all the zeros of f .
- (b) Find $f'(1)$.
- (c) Find $f'(-1)$.
- (d) Find the range of f .

(75-5)

4. The line $x = c$, where $c > 0$, intersects the cubic $y = 2x^3 + 3x^2 - 9$ at point P and the parabola $y = 4x^2 + 4x + 5$ at point Q .

- (a) If a line tangent to the cubic at point P is parallel to the line tangent to the parabola at point Q , find the value of c where $c > 0$.
- (b) Write the equations of the two tangent lines described in (a).

(90-6)

5. Let f be the function that is given by $f(x) = \frac{ax+b}{x^2-c}$ and that has the following properties.

(i) The graph of f is symmetric with respect to the y -axis.

(ii) $\lim_{x \rightarrow 2^+} f(x) = +\infty$

(iii) $f'(1) = -2$

(a) Determine the values of a , b , and c .

(b) Write an equation for each vertical and each horizontal asymptote of the graph of f .

(c) Sketch the graph of f in the xy -plane.

(95-1)

6. Let f be the function given by $f(x) = \frac{2x}{\sqrt{x^2+x+1}}$

(a) Find the domain of f . Justify your answer.

(b) Sketch the graph of f in a viewing window $[-5,5], [-3,3]$.

(c) Write an equation for each horizontal asymptote of the graph of f .

(d) Find the range of f . Use $f'(x)$ to justify your answer. Note: $f'(x) = \frac{x+2}{(x^2+x+1)^{\frac{3}{2}}}$

(86-4)

7. Let f be the function defined as follows.

$$f(x) = \begin{cases} |x-1| + 2, & \text{for } x < 1 \\ ax^2 + bx, & \text{for } x \geq 1, \text{ where } a \text{ and } b \text{ are constants} \end{cases}$$

(a) If $a = 2$ and $b = 3$, is f continuous for all x ? Justify your answer.

(b) Describe all values of a and b for which f is a continuous function.

(c) For what values of a and b is f both continuous and differentiable?

(85-1)

8. Let f be the function given by $f(x) = \frac{2x-5}{x^2-4}$

(a) Find the domain of f .

(b) Write an equation for each vertical and each horizontal asymptote for the graph of f .

(c) Find $f'(x)$.

(d) Write an equation for the line tangent to the graph of f at the point $(0, f(0))$.