AP PROBLEM SET #3 DERIVATIVES: IMPLICIT

(94-3)

1. Consider the curve defined by $x^2 + xy + y^2 = 27$.

(a) Write an expression for the slope of the curve at any point (x,y).

(b) Determine whether the lines tangent to the curve at the x-intercepts of the curve are parallel. Show the analysis that leads to your conclusion.

(c) Find the points on the curve where the lines tangent to the curve are vertical.

(2004-4)

2. Consider the curve defined by $x^2 + 4y^2 = 7 + 3xy$.

(a) Show that $\dfrac{dy}{dx} = \dfrac{3y - 2x}{8y - 3x}$.

(b) Show that there is a point P with x-coordinate 3 at which the line tangent to the curve at P is horizontal. Find the y-coordinate of P.

(c) Find the value of $\dfrac{d^2y}{dx^2}$ at the point P found in part (b). Does the curve have a local minimum, a local maximum, or neither at the point P? Justify your answer.

(92-4)

3. Consider the curve defined by the equation $y + \cos y = x + 1$ for $0 \le y \le 2\pi$.

(a) Find $\dfrac{dy}{dx}$ in terms of y.

(b) Write an equation for each vertical tangent to the curve.

(c) Find $\dfrac{d^2y}{dx^2}$ in terms of y.

(2000-5)

4. Consider the curve given by $xy^2 - x^3y = 6$.

(a) Show that $\dfrac{dy}{dx} = \dfrac{3x^2y - y^2}{2xy - x^3}$

(b) Find all points on the curve whose x-coordinate is 1, and write an equation for the tangent line at each of these points.

(c) Find the x-coordinate of each point on the curve where the tangent line is vertical.

(95-3)

5. Consider the curve defined by $-8x^2 + 5xy + y^3 = -149$.

(a) Find $\frac{dy}{dx}$.

(b) Write an equation for the line tangent to the curve at the point (4, −1).

(c) There is a number k so that the point (4.2, k) is on the curve. Using the tangent line found in part (b), approximate the value of k.

(d) Write an equation that can be solved to find the actual value of k so that the point (4.2, k) is on the curve.

(e) Solve the equation found in part (d) for the value of k.

(2015-6)

6. Consider the curve given by the equation $y^3 - xy = 2$. It can be shown that $\frac{dy}{dx} = \frac{y}{3y^2 - x}$.

(a) Write an equation for the line tangent to the curve at the point (−1, 1).

(b) Find the coordinates of all points on the curve at which the line tangent to the curve at that point is vertical.

(c) Evaluate $\frac{d^2y}{dx^2}$ at the point on the curve where $x = -1$ and $y = 1$.