

AP CALCULUS PROBLEM SET 4 ANSWER KEY

1. a) $x < 0$ or $x > 1$

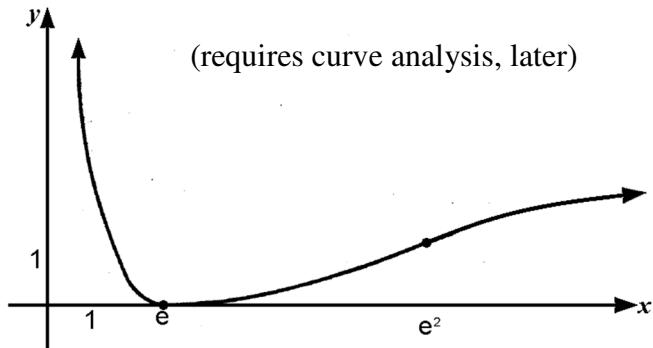
b) $f'(-1) = -\frac{1}{2}$

c) $f^{-1}(x) = \frac{e^x}{e^x - 1}$

2. a) $f(g(x)) = (1 - \ln x)^2 = h(x)$

b) $h'(x) = -\frac{2}{x}(1 - \ln x)$

c) $h''(x) = \frac{-2(\ln x - 2)}{x^2}$

d) 
(requires curve analysis, later)

3. a) even

b) $\left(-\infty, -\frac{1}{\sqrt{2}}\right]$ or $\left[\frac{1}{\sqrt{2}}, \infty\right)$

c) $[1, \infty)$

d) $f'(x) = \frac{(2x \ln 5) 5^{\sqrt{2x^2-1}}}{\sqrt{2x^2-1}}$

4. a) $f(x)$ is defined for all $x \neq 0$

b) $x = \pm e$

c) $y = 2x - 4$

5. a) $m = \frac{1}{e}$

b) Show that $\frac{1}{e}(x) - \ln x, x > 0$
is always positive

$$f(x) = \frac{1}{e}(x) - \ln x = \frac{x}{e} - \ln x$$

$$f'(x) = \frac{1}{e} - \frac{1}{x} = 0 \text{ when } x = e$$

Sign Analysis shows absolute minimum
at $x = e$,

$$f(e) = 0, \therefore \frac{x}{e} - \ln x \geq 0$$

c) from b), $\frac{x}{e} \geq \ln x$

$$x \geq e \ln x$$

$$x \geq \ln x^e$$

$$e^x \geq x^e$$

6. a) $\lim_{x \rightarrow 0^-} (1 - 2 \sin x) = 1$

$$\lim_{x \rightarrow 0^+} e^{-4x} = 1$$

$$f(0) = 1$$

$$\therefore \lim_{x \rightarrow 0} f(x) = f(0)$$

b) $f'(x) = \begin{cases} -2 \cos x & \text{for } x < 0 \\ -4e^{-4x} & \text{for } x > 0 \end{cases}$

$$f'(x) = -3 \text{ for } x = -\frac{1}{4} \ln \left(\frac{3}{4} \right)$$

c) $\frac{13}{8} - \cos(-1) - \frac{1}{8} e^{-4}$