

AP CALCULUS PROBLEM SET #5 MOTION ANSWER KEY

1. a) $x(t) = t^3 - t^2 - t + 3$

b) $t = 1.786$

c) 17

2. a) $a(0) = -4$

b) $v(2) = \frac{-2}{e^2}$

c) $\frac{4e^4 - 10}{e^5}$

3. a) $3 < t < 9$

b) $\int_0^6 |v(t)| dt$

c) $a(4) = -\frac{\sqrt{3}\pi}{12}$

Speed is increasing at $t = 4$ because velocity and acceleration are both negative

d)
$$\begin{aligned} x(4) &= -2 + \int_0^4 \cos\left(\frac{\pi}{6}t\right) dt \\ &= -2 + \left[\frac{6}{\pi} \sin\left(\frac{\pi}{6}t\right) \right]_0^4 \\ &= -2 + \frac{3\sqrt{3}}{\pi} \end{aligned}$$

4. a) $0 < t < 1$ and $3 < t < 6$

b) $0 < t < 1$ and $3 < t < 4$

c) $v = \frac{dp}{dt} \Big|_{t=3} < 0$

$a = \frac{d^2p}{dt^2} \Big|_{t=3} = \frac{\pi^2}{8\sqrt{2}} > 0$

Particle is slowing down at time $t = 3$

d) $\frac{1}{2} \int_1^3 |p(t) - r(t)| dt$

5. a) Particle is furthest to the left at $t = 3$
When its position is $x(3) = -10$

b) There are 3 values of t for which the particle is at $x(3) = 8$

5. c) Speed is decreasing on the interval $(2, 3)$ since $v < 0$ and $\frac{dv}{dt} > 0$

d) Acceleration is negative on the intervals $(0, 1)$ and $(4, 6)$ since $\frac{dv}{dt} < 0$

6. a) A: $\frac{20}{3}$ m/s

B: $\frac{48}{7}$ m/s

b) A: $\frac{10}{3}$ m/s²

B: $\frac{72}{49}$ m/s²

c) A: 85 m
B: 83.336 m

7. a) yes, $a(2) = 15$

d) $t = 12$

c) 115 ft/sec

d) velocity is never zero

8. a) up, $v(1.5) > 0$

b) $a(1.5) = -2.048$

c) $y(2) = 3.826$

d) 1.173

9. a) $t = 1$ or $\frac{2}{3}$

b) $\frac{2}{3} < t < 1$

c) $t = \frac{5}{6}$