

## AP CALCULUS PROBLEM SET # 9 INTEGRATION I ANTIDERIVATIVES

(74-4)

1. Let  $f$  be a function defined for all  $x > -5$ , and having the following properties.

(i)  $f''(x) = \frac{1}{3\sqrt{x+5}}$  for all  $x$  in the domain of  $f$ .

- (ii) The line tangent to the graph of  $f$  at  $(4, 2)$  has an angle of inclination of  $45^\circ$ .

Find an expression for  $f(x)$ .

(88-6)

2. Let  $f$  be a differentiable function, defined for all real numbers  $x$ , with the following properties:

(i)  $f'(x) = ax^2 + bx$

(ii)  $f'(1) = 6$  and  $f''(1) = 18$

(iii)  $\int_1^2 f(x)dx = 18$

Find  $f(x)$ . Show your work.

(89-BC1)

3. Let  $f$  be a function such that  $f''(x) = 6x + 8$ .

- (a) Find  $f(x)$  if the graph of  $f$  is tangent to the line  $3x - 2$  at the point  $(0, -2)$ .

- (b) Find the average value of  $f(x)$  on the closed interval  $[-1, 1]$ .

(2011(B)-4)

4. Consider a differentiable function  $f$  having domain all positive real numbers, and for which it is known that  $f'(x) = (4-x)x^{-3}$  for  $x > 0$ .

- (a) Find the  $x$ -coordinate of the critical point of  $f$ . Determine whether the point is a relative maximum, a relative minimum, or neither for the function  $f$ . Justify your answer.

- (b) Find all intervals on which the graph of  $f$  is concave down. Justify your answer.

- (c) Given that  $f(1) = 2$ , determine the function  $f$ .

(2003-6)

5. Let  $f$  be the function defined by  $f(x) = \begin{cases} \sqrt{x+1} & \text{for } 0 \leq x \leq 3 \\ 5-x & \text{for } 3 < x \leq 5 \end{cases}$

(a) Is  $f$  continuous at  $x = 3$ ? Explain why or why not.

(b) Find the average value of  $f(x)$  on the closed interval  $0 \leq x \leq 5$ .

(c) Suppose the function  $g$  is defined by  $g(x) = \begin{cases} k\sqrt{x+1} & \text{for } 0 \leq x \leq 3 \\ mx+2 & \text{for } 3 < x \leq 5, \end{cases}$  where  $k$  and  $m$  are constants. If  $g$  is differentiable at  $x = 3$ , what are the values of  $k$  and  $m$ ?

(1990BC-6)

6. Let  $f$  and  $g$  be differentiable functions with the following properties:

(i)  $g(x) = A - f(x)$  where  $A$  is a constant

(ii)  $\int_1^2 f(x)dx = \int_2^3 g(x)dx$

(iii)  $\int_2^3 f(x)dx = -3A$

(a) Find  $\int_1^3 f(x)dx$  in terms of  $A$ .

(b) Find the average value of  $g(x)$  in terms of  $A$ , over the interval  $[1, 3]$ .

(c) Find the value of  $k$  if  $\int_0^1 f(x+1)dx = kA$ .