2.3 Different Number Systems

We live in a world that uses the base 10 number system. Each digit represents a power of 10. Take, for example, the number 1904. You intuitively understand the meaning of each digit because you have grown up with this system:

Digit	1	9	0	4
Place Value as a Power	10^3	10^2	10^1	10^0
Place Value as a Number	1000	100	10	1
Value of Digit	1 x 1000 = 1000	9 x 100 = 900	0 x 10 = 0	4 x 1 = 4

In base 10 numbers, each digit is worth an increasingly larger power of 10 as the digits move from right to left.

Computers do not store numbers in base 10. Because of the limitations of computer hardware, it is much easier for computers to store numbers in base 2, also known as **binary**. In base 2, each digit is worth an increasingly larger power of 2 as the digits move from right to left.

Take for example 1010110. What base 10 number does this represent?

Digit	1	0	1	0	1	1	0
Place Value as a Power	2^6	2^5	2^4	2^3	2^2	2^1	2^0
Place Value as a Number	64	32	16	8	4	2	1
Value of Digit	1x64=64	0x32=0	1x16=16	0x8=0	1x4=4	1x2=2	0x1=0

So, 1010110 in base 10 would be 64 + 16 + 4 + 2 = 86!

How to Convert from Decimal to Binary

Let's convert the base 10 number 150 into base 2. First, find the largest power of 2 that will fit into 150. Put a 1 in that spot. Then, move down to each lower place and fill in a 1 at each place if it will fit. Try it yourself right now:

Digit								
Place Value as a Power	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
Place Value as a Number	128	64	32	16	8	4	2	1
Value of Digit								

The answer is on the next page.

The correct answer is 10010110

Each digit is stored in a unit of memory called a **bit**. Because there can only be 2 digits, physical memory can store these values using an kind of electronic switch. If the switch is off, this represents 0. If the switch is on, this represents 1. 8 bits make a **byte**, which is usually how memory is measured. For example, a gigabyte contains 1,073,741,824 bytes (often rounded to 1 billion).

Question 1: Count from 0 to 15 in base 2

Question 2: Convert the following base 10 numbers to binary:

a. 96 b. 255 c. 499

Question 3: Convert the following base 2 numbers to base 10:

a. 11011 c. 1010101 d. 100001

Question 4: How many different numbers can be stored in a byte of memory?

Hexadecimal Numbers

Hexadecimal numbers are in base 16. As a result, we need some extra digits:

Dec	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Hex	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
Bin	0	1	10	11	100	101	110	111	1000	1001	1010	1011	1100	1101	1110	1111

Convert the hex number A3F into decimal:

Digit	A	3	F
Place Value as a Power	16^2	16^1	16^0
Place Value as a Value	256	16	1
Value of Digit	10 x 256 = 2560	3 x 16 = 48	15 x 1 = 15

So, A3F in base 10 is 2560 + 48 + 16 = 2623

Converting Hex to Binary

This is what Hex is very useful for. Each digit in hex converts to a neat set of 4 digits in binary. To convert from Hex to Binary, just convert each digit into a 4 digit binary number

Example: convert the hex number A3F to binary:

Digit in Hex	A	3	F
Digit in Binary	1010	0011	1111

Thus, A3F = 1010 0011 1111 in Binary.

Converting Binary to Hex

Converting from Binary to Hex is equally simple. Just break your binary number into groups of 4 digits, and convert each of those groups of digits separately.

Example: Convert 1000 1101 0010 1101 into base 16.

Digit in Binary	1000	1101	0010	1101
Digit in Decimal	8	13	2	13
Digit in Hex	8	D	2	D

So, 1000 1101 0010 1101 is 8D2D in base 16.

Question 5: Convert the following base 16 numbers into base 10

a. 10 b. FF c. ABC

Question 6: Convert the following base 16 numbers into base 2:

a. FA2 b. CAB c. 9BF

Question 7: Convert the following base 2 numbers into hex:

a. 1001 1100 0101 1001 b. 10010

Question 8: The following numbers are in different bases. Convert each one back into base 10.

a. Base 3: 1202 b. Base 9: 1538

Question 9: Convert each base 10 number into the indicated number system

c. 66 to base 7 d. 100 to base 4